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EDUCATION
REPUBLIC OF SOUTH AFRICA

PREPARATORY EXAMINATIONS

VOORBEREIDENDE EKSAMEN

2018

MARKING GUIDELINES /

NASIENRIGLYNE

MATHEMATICS / *WISKUNDE*

(PAPER 2 / *VRAESTEL 2*) (10612)

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GAUTENG DEPARTMENT OF EDUCATION

GAUTENGSE DEPARTEMENT VAN ONDERWYS

PROVINCIAL EXAMINATION / *PROVINSIALE EKSAMEN*

MATHEMATICS / *WISKUNDE*
(PAPER 2 / *VRAESTEL 2*)

MARKING GUIDELINES / *NASIENRIGLYNE*

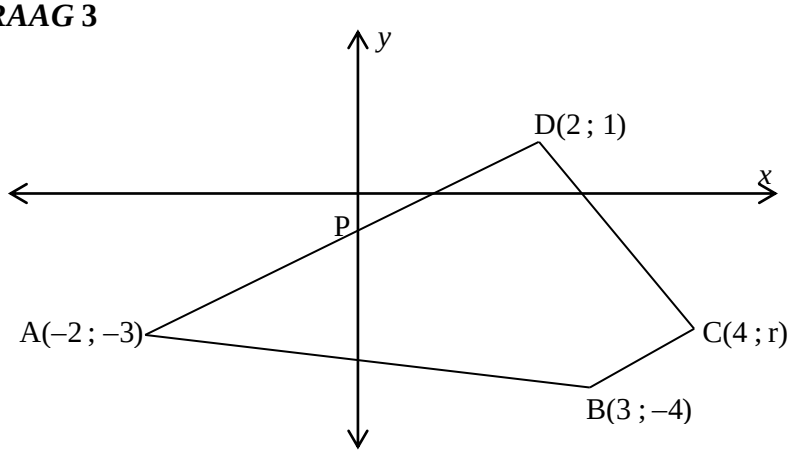
QUESTION / *VRAAG* 1

1.1	$a = 24$	✓✓ $a = 24$ (2)
1.2	100	✓ 100 (1)
1.3	$50 \le x < 60$	✓ answer (1)
1.4.1	$\bar{x} = \frac{24\times15+6\times25+8\times35+28\times45+34\times55}{100}$ $= 39,2$ Answer only 3/3	✓ numerator ✓✓ 39,2 (3)
1.4.2	$100 - 34$ $= 66$	✓ 34 ✓ 66 (2)
		[9]

QUESTION / *VRAAG* 2

2.1	$\sigma = 6,47$	✓ answer (1)
2.2	$\bar{x} = 14,5$ One standard deviation above the mean: $14,5 + 6,47$ $= 20,97$ Therefore a student needed to work for **21** hours	✓ $\bar{x} = 14,5$ ✓ 20,97 ✓ 21 hours (3)
2.3	$y = 454,38 + 131,42x$	✓ $a = 454,38$ ✓ $b = 131,42$ ✓ equation / *vergelyking* (3)
2.4	Payment when $x = 11,5$ / *Betaling wanneer* $x = 11,5$ $y = 454,38 + 131,42x$ **OR / OF** $= 454,38 + 131,42(11,5)$ by using the calculator $= \text{R}\,1965,71$ R 1 965,73 2 / 2 marks	✓ subst x = 11,5 / *vervang* x = 11.5 ✓ R1965,71 (2)
2.5	$(23\,;\,2700)$ The student could have done some of the work incorrectly and needed to redo work without any payment ***Any logical reason can be accepted***	✓ $(23\,;\,2700)$ ✓ reason (2)
		[11]

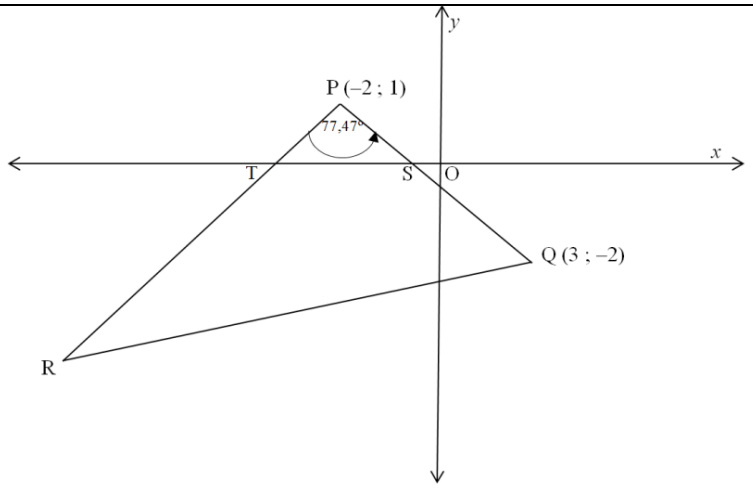
QUESTION / *VRAAG* 3



3.1.1	$m_{AD} = \frac{1-(-3)}{2-(-2)}$ $= 1$ $m_{AD} = m_{BC} = 1 \quad [AD \parallel BC]$ $1 = \frac{r-(-4)}{4-3}$ $1 = \frac{r+4}{1}$ $r + 4 = 1$ $r = -3$ **OR / OF** $m_{AD} = m_{BC} \quad [AD \parallel BC]$ $\frac{1+3}{2+2} = \frac{r+4}{4-3}$ $r + 4 = 1$ $r = -3$	✓ $m_{AD} = 1$ ✓ $\frac{r-(-4)}{4-3}$ ✓ $r + 4 = 1$ ✓ value of r / *waarde van r* ✓ $m_{AD} = 1$ ✓ $\frac{r-(-4)}{4-3}$ ✓ $r + 4 = 1$ ✓ value of r / *waarde van r* (4)
3.1.2	Trapezium / *Trapesium*	✓ answer / *antwoord* (1)
3.1.3	$P\left(\frac{-2+2}{2}; \frac{-3+1}{2}\right)$ $= P(0 ; -1)$ **OR / OF** $y - 1 = 1(x - 2)$ $y = x - 1$ $P(0 ; -1)$	✓ $x = 0$ ✓ $y = -1$ (2)
3.1.4	$m_{PB} = \frac{-1-(-4)}{0-3}$ $= -1$ $m_{AD} \times m_{PB} = 1 \times -1$ $= -1$ $\therefore BP \perp AD$	✓ $m_{PB} = -1$ ✓ product = –1 (2)

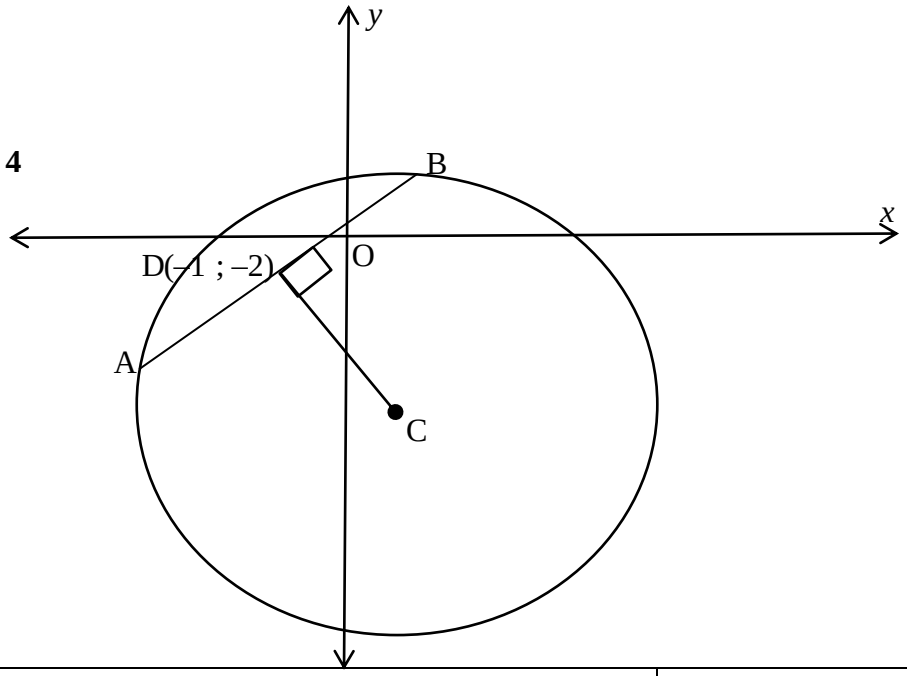
3.1.5	AB is a diameter / *middellyn* $[A\hat{P}B = 90^\circ]$ $AB = \sqrt{26}$ $r = \frac{\sqrt{26}}{2}$ $\therefore r^2 = \frac{26}{4}$ or $\frac{13}{2}$ midpoint of AB / *middelpunt van AB* $= \left(\frac{3-2}{2}\ ;\ \frac{-4-3}{2}\right)$ $= \left(\frac{1}{2}\ ;\frac{-7}{2}\right)$ $\therefore \left(x-\frac{1}{2}\right)^2 + \left(y+\frac{7}{2}\right)^2 = \frac{13}{2}$ **OR / *OF*** midpoint of AB / *middelpunt van AB* $= \left(\frac{3-2}{2};\frac{-4-3}{2}\right)$ $= \left(\frac{1}{2};\frac{-7}{2}\right)$ $r^2 = \left(\frac{1}{2}+2\right)^2 + \left(-\frac{7}{2}+3\right)^2$ or / *of* $r^2 = \left(0-\frac{1}{2}\right)^2 + \left(-1+\frac{7}{2}\right)^2$ $= \frac{25}{4}+\frac{1}{4}$ $= \frac{1}{4}+\frac{25}{4}$ $= \frac{13}{2}$ $= \frac{13}{2}$ $\therefore \left(x-\frac{1}{2}\right)^2 + \left(y+\frac{7}{2}\right)^2 = \frac{13}{2}$	✓ $r = \frac{\sqrt{26}}{2}$ ✓ $r^2 = \frac{13}{2}$ ✓ $\frac{1}{2}$ ✓ $-\frac{7}{2}$ ✓ equation / *vergelyking* ✓ $\frac{1}{2}$ ✓ $-\frac{7}{2}$ ✓ substitution into distance formula / *vervang in afstandformule* ✓ $r^2 = \frac{13}{2}$ ✓ equation / *vergelyking* (5)
3.1.6	$x^2 + y^2 - 2x\cos\theta - 4y\cos\theta = -2$ $x^2 - 2x\cos\theta + y^2 - 4y\cos\theta = -2$ $(x-\cos\theta)^2 + (y-2\cos\theta)^2 = -2 + \cos^2\theta + 4\cos^2\theta$ $r^2 = -2 + \cos^2\theta + 4\cos^2\theta$ $= -2 + 5\cos^2\theta$ For any value of θ the maximum of $\cos^2\theta = 1$ $r^2 = -2 + 5$ Maximum value of $r = \sqrt{3}$	✓ $(x-\cos\theta)^2 + (y-2\cos\theta)^2$ ✓ $-2 + \cos^2\theta + 4\cos^2\theta$ ✓ $r^2 = -2 + 5\cos^2\theta$ ✓ max of $\cos^2\theta = 1$ ✓ $r = \sqrt{3}$ (5)

3.2



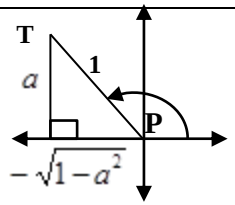
3.2.1	$m_{PQ} = \frac{-2-1}{3+2}$ $= -\frac{3}{5}$ $y-1 = -\frac{3}{5}(x+2)$ or / *of* $-2 = -\frac{3}{5}(3)+c$ $5y-5 = -3x-6$ $-2 = -\frac{9}{5}+c$ $3x+5y+1=0$ $c = -\frac{1}{5}$ $y = -\frac{3}{5}x - \frac{1}{5}$ $3x+5y+1=0$	✓gradient of PQ / *gradiënt van PQ* ✓sub P, Q and / *en* m ✓equation / *vergelyking* (correct form) (3)
3.2.2	$\tan P\hat{S}O = m_{PQ}$ $= -\frac{3}{5}$ $P\hat{S}O = 180^o - 30,96^o$ $= 149,04^o$ $P\hat{T}S = 149,04^o - 77,47^o$ [ext ∠ of Δ]/ $= 71,57^o$ *[buite ∠ van Δ]* $m_{PR} = \tan 71,57^o$ $= 3$ $y-1 = 3(x+2)$ or/*of* $1 = 3(-2)+c$ $y = 3x+7$ $y = 3x+7$	✓ $\tan P\hat{S}O = -\frac{3}{5}$ ✓ $P\hat{S}O = 149,04^o$ ✓ $P\hat{T}S = 71,57^o$ ✓ $m_{PR} = 3$ ✓sub P(–2;1) and / *en* *m* ✓equation /*vergelyking* (correct form) (6) **[28]**

QUESTION / *VRAAG* 4



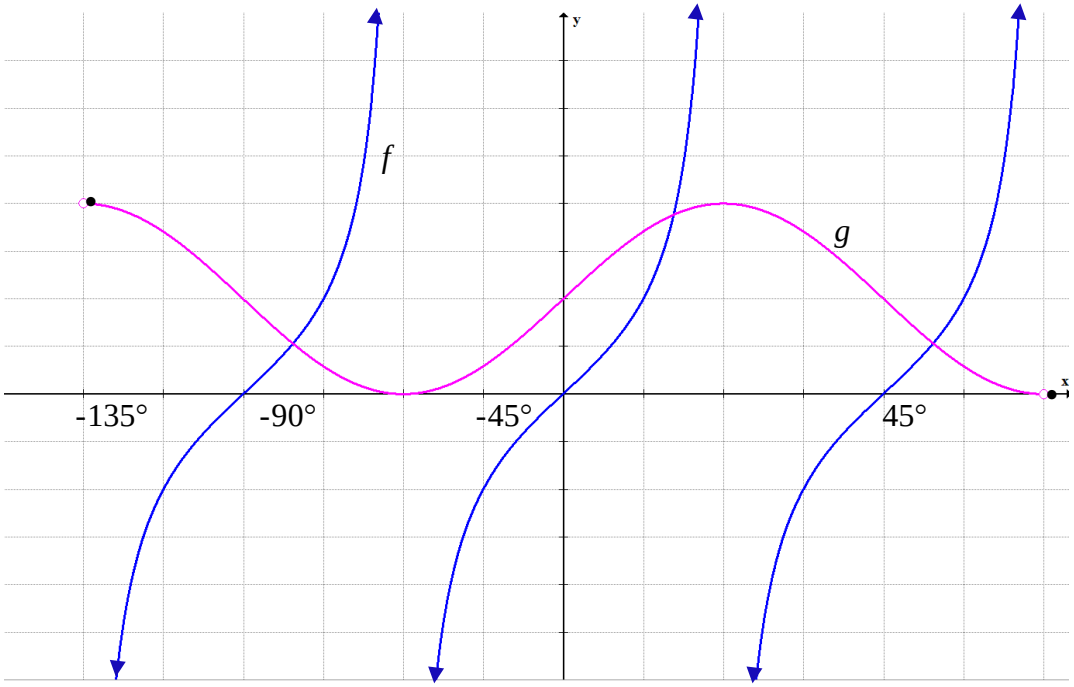
4.1	$x^2 + y^2 + 6y - 4x = 12$ $(x-2)^2 + (y+3)^2 = 12 + 4 + 9$ $(x-2)^2 + (y+3)^2 = 25$ $C(2\ ;-3)$ Answer only 3 / 3	✓ $(x-2)^2 + (y+3)^2$ ✓✓ $C(2\ ;-3)$ (3)
4.2	$r = 5$	✓ $r = 5$ (1)
4.3	$DC^2 = (2+1)^2 + (-3+2)^2$ $= 9 + 1$ $=10$ $BC^2 = 25$ (radius) $DB^2 = BC^2 - DC^2$ (Pyth) $= 25 - 10$ $= 15$ $DB = \sqrt{15}$ $AB = 2\sqrt{15}$ or / *of* 7,75	✓sub into distance formula / *vervang in afstandformule* ✓$DC^2 = 10$ ✓sub into / *in* Pythagoras ✓$DB = \sqrt{15}$ ✓ $AB = 2\sqrt{15}$ or / *of* 7,75 (5)
4.4	$\text{Area} = \frac{1}{2}(AB)(DC)$ $= \frac{1}{2}\left(2\sqrt{15}\right)\left(\sqrt{10}\right)$ $= 5\sqrt{6}$ / 12,25	✓ area formula / *oppervlakte formule* ✓ substitution / *vervanging* (ca from / *van* 4.2) ✓ answer / *antwoord* (3) **[12]**

QUESTION / *VRAAG* 5

5.1	$\dfrac{\sin x.\sin(90°+y)-\cos x.\sin(180°+y)}{\cos x.\cos(y-360°)+\sin(-x)\sin y}$ $=\dfrac{\sin x.\cos y-\cos x.(-\sin y)}{\cos x.\cos y+(-\sin x).\sin y}$ $=\dfrac{\sin x.\cos y+\cos x.\sin y}{\cos x.\cos y-\sin x.\sin y}$ $=\dfrac{\sin(x+y)}{\cos(x+y)}$ $=\tan(x+y)$	✓ $\cos y$ ✓ $-\sin y$ ✓ $\cos y$ ✓ $-\sin x$ ✓ $\dfrac{\sin(x+y)}{\cos(x+y)}$ ✓ $\tan(x+y)$ (6)
5.2.1	$\cos(A+B)$ $=\cos(A-(-B))$ $=\cos A\cos(-B)+\sin(A)\sin(-B)$ $=\cos A\cos B-\sin A\sin B$ $=$ RHS	✓ $\cos(A-(-B))$ ✓ subst $B\rightarrow -B$ (2)
5.2.2 (a)	 $x^2+y^2=r^2$ $x^2=-\sqrt{1-a^2}$ $T\left(-\sqrt{1-a^2}\,;a\right)$	✓ Pythagoras ✓ $T\left(-\sqrt{1-a^2}\,;a\right)$ (2) Answer only 2/2
5.2.2 (b)	$R\left(-\sqrt{1-a^2}\,;-a\right)$	✓ ✓ $R\left(-\sqrt{1-a^2}\,;-a\right)$ (2)
5.2.2 (c)	$\cos(P+Q)=\cos P\cos Q-\sin P\sin Q$ $=\left(-\sqrt{1-a^2}\right)\left(-\sqrt{1-a^2}\right)-(a)(-a)$ $=1-a^2+a^2$ $=1$	✓ $\left(-\sqrt{1-a^2}\right)\left(-\sqrt{1-a^2}\right)$ $-(a)(-a)$ ✓ 1 (2)
5.2.2 (d)	$\cos 360°=1$ $\hat{P}+\hat{Q}=360°$; $\hat{P}>90^{\circ\circ}$; $\hat{Q}>180^{\circ\circ}$	✓ answer (1)

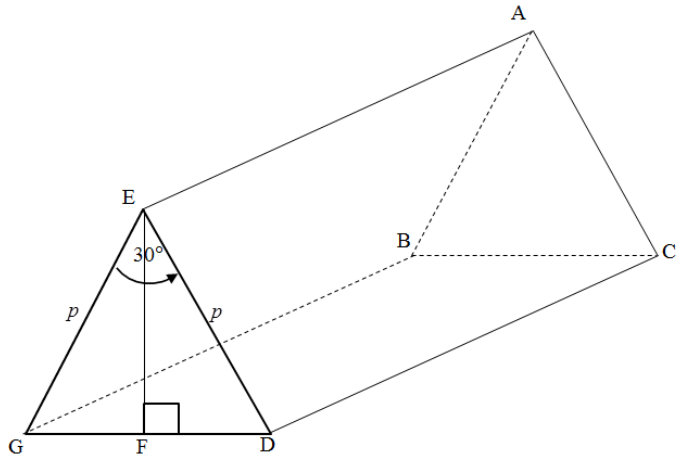
5.3.1	$-1 \le d \le 1$ or / *of* $[-1\ ;\ 1]$	✓✓ answer / *antwoord* (2)
5.3.2	$\cos\theta = \frac{1}{\cos\theta} + \frac{5}{6}$ $6\cos^2\theta = 6 + 5\cos\theta$ $6\cos^2\theta - 5\cos\theta - 6 = 0$ $(3\cos\theta + 2)(2\cos\theta - 3) = 0$ $\cos\theta = -\frac{2}{3}$ or $\cos\theta = \frac{3}{2}$ no solution/geen oplossing $\theta = \pm 131{,}81° + k.360°\ ;\ k \in Z$ **OR / *OF*** $\theta = 180° - 48{,}19° + k.360°$ or $\theta = 180° + 48{,}19° + k.360°$ $= 131{,}81° + k.360°\ ;\ k \in Z$ $= 228{,}19° + k.360°;\ k \in Z$	✓ $6\cos^2\theta - 5\cos\theta - 6 = 0$ ✓factors / *faktore* ✓ both solutions ✓ choosing $\cos\theta = -\frac{2}{3}$ ✓ $\theta = 131{,}81° + k.360°$ ✓ $\theta = -131{,}81° + k.360°$ **OR / *OF*** $228{,}19° + k.360°$ If $k \in Z$ is omitted then subtract one mark / *As $k \in Z$ weggelaat is, trek een punt af* (6)
		[23]

QUESTION / *VRAAG* 6

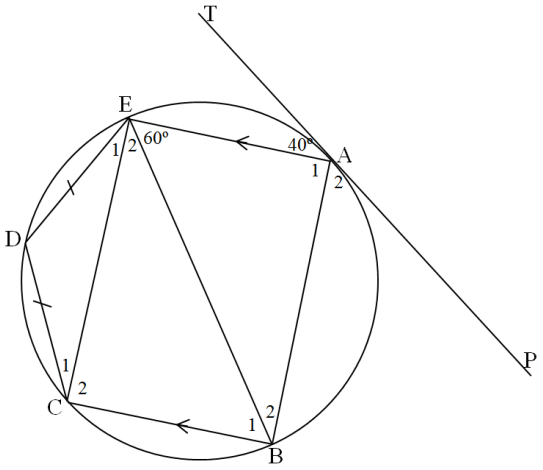


6.1	$x = -45°$	✓ $x = -45°$ (has to be an equation x =..) (1)
6.2	$h(x) = \dfrac{\sin x - 2\sin^3 x}{2\sin^2 x.\cos x}$ $= \dfrac{\sin x(1-2\sin^2 x)}{2\sin^2 x.\cos x}$ $= \dfrac{(1-2\sin^2 x)}{2\sin x.\cos x}$ $= \dfrac{\cos 2x}{\sin 2x}$ $= \dfrac{1}{\tan 2x}$ $= \dfrac{1}{f(x)}$	✓ $\sin x(1-2\sin^2 x)$ ✓ $\cos 2x$ ✓ $\sin 2x$ ✓ $\dfrac{1}{f(x)}$ (4)
6.3	$p(x) = 1+\sin 2(x+45°)$ $= 1+\sin(2x+90°)$ $= 1+\cos 2x$ Answer only 3/3	✓ $1+\sin 2(x+45°)$ ✓ $\sin(2x+90°)$ ✓ $1+\cos 2x$ (3)
6.4	$(\tan 2x).(-1-\sin 2x) \le 0$ $(\tan 2x).(1+\sin 2x) \ge 0$ $-90° \le x < -45°$ or $[-90° ; -45°)$	✓ $(\tan 2x).(1+\sin 2x) \ge 0$ ✓ $-90° \le x$ ✓ $x < -45°$ (3)
		[11]

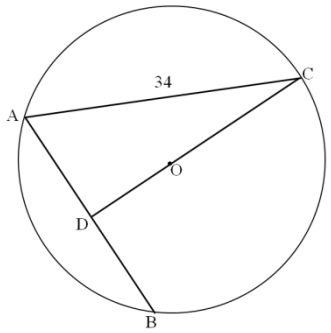
QUESTION / *VRAAG* 7

7.1	 $GD^2 = EG^2 + ED^2 - 2EG.ED\cos G\hat{E}D$ $= p^2 + p^2 - 2p.p\cos 30°$ $= 2p^2 - 2p^2\left(\frac{\sqrt{3}}{2}\right)$ $= 2p^2 - \sqrt{3}p^2$ $= p^2(2-\sqrt{3})$	✓ correct substitution in cos-rule / *korrekte vervanging in cos-reël* ✓ $\frac{\sqrt{3}}{2}$ ✓ simplification / *vereenvoudiging* $2p^2 - \sqrt{3}p^2$ (3)
7.2	In ΔCDG $\frac{CD}{GD} = \tan C\hat{G}D$ $CD = GD.\tan 60°$ $= \sqrt{p^2(2-\sqrt{3})}.\sqrt{3}$ $= \sqrt{p^2(2-\sqrt{3}).3}$ $= p\sqrt{6-3\sqrt{3}}$	✓ $\frac{CD}{GD} = \tan C\hat{G}D$ ✓ $\sqrt{p^2(2-\sqrt{3})}$ ✓ $\sqrt{3}$ (3)
		[6]

QUESTION / *VRAAG* 8

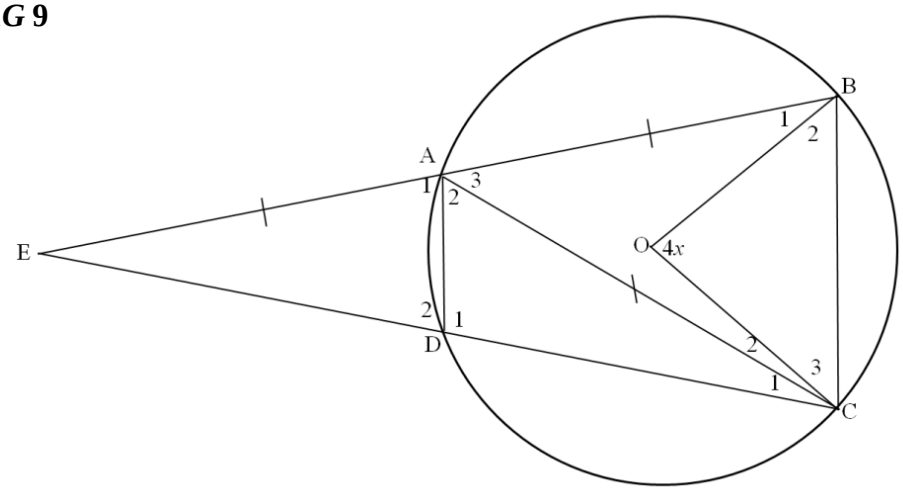


8.1	EABC EBCD	✓ EABC ✓ EBCD (2)
8.2.1	$\hat{B}_2 = 40^o$ [tan chord theorem / *∠ tussen raaklyn en koord*]	✓S✓R (2)
8.2.2	$\hat{B}_1 = 60^o$ [alt $\angle^s$; AE ∥ BC] / *verw. binne* $\angle^e$; AE ∥ BC	✓S ✓R (2)
8.2.3	$\hat{D} = 120^o$ [opp $\angle^s$ of a cyclic quad / *teenoorst.* $\angle^e$ *van kvh*]	✓S✓R (2)
8.2.4	$\hat{E}_1 = \hat{C}_1$ [$\angle^s$ opp equal sides / $\angle^e$ *teenoor gelyke sye*] $\hat{E}_1 = 30^o$ [sum of $\angle^s$ of a Δ / *som* $\angle^e$ *van Δ* / $\angle^e$ *van Δ*]	✓S ✓S ✓R (3)



8.3	AD = 20 $A\hat{D}C = 90°$ [line from centre to midpoint of chord / *midpt. ⊙; midpt. koord* ***OF*** *lyn van midpt ⊙ na midpt van koord*] $\sin\hat{C} = \frac{20}{34}$ $\sin\hat{C} = 0{,}588$ $\hat{C} = 36{,}03°$ **Accept 36°**	✓AD = 20 ✓ R ✓ $\sin\hat{C} = \frac{20}{34}$ ✓ $\hat{C} = 36°$ (4)
		[15]

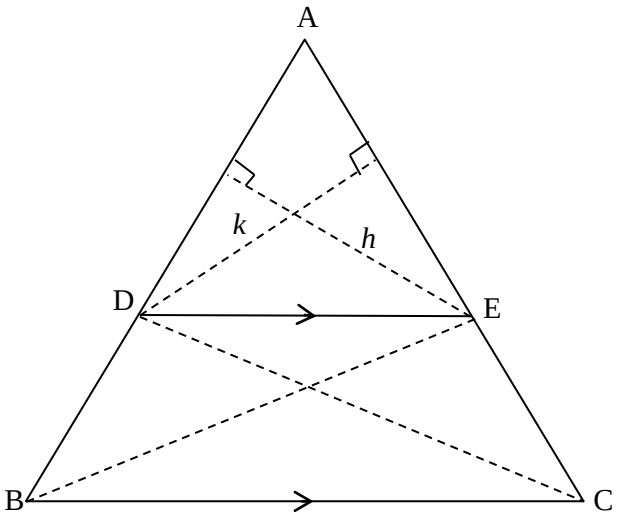
QUESTION / *VRAAG* 9



9.1	In ΔOBC $\hat{B}_2 = \hat{C}_3$ ($\angle$'s opposite equal radii / $\angle^e$ *teenoor gelyke radii*) $\hat{B}_2 = 90° - 2x$ (sum of $\angle$'s of a Δ // *som* $\angle^e$ *van* Δ / $\angle^e$ *van* Δ)	✓ S / R ✓ $\hat{B}_2 = 90° - 2x$ (2)
9.2	$\hat{A}_3 = 2x$ ($\angle$ at centre = $2 \times \angle$ at circumference *midpts* $\angle = 2 \times$ *omtreks* $\angle$) $\hat{A}_3 = \hat{C}_1 + \hat{E}$ (ext. $\angle$ of a Δ / *buite* $\angle$ *van* Δ) but/*maar* AB = AC = AE (given /*gegee*) $\hat{C}_1 = \hat{E}$ ($\angle$'s opp equal sides / $\angle^e$ *teenoor gelyke sye*) $\therefore \hat{E} = x$	✓ S ✓ R ✓ S ✓ S ✓ $\hat{E} = x$ (5)
9.3	$\hat{B}_1 + \hat{B}_2 = \hat{C}_2 + \hat{C}_3$ ($\angle$'s opp equal sides / $\angle^e$ *teenoor gelyke sye*) $\hat{B}_1 = \hat{C}_2 = 180° - (2x + 90° - 2x + 90° - 2x)$ (sum of $\angle$'s of a Δ / som $\angle^e$ *van* Δ / $\angle^e$ *van* Δ) $\therefore \hat{C}_2 = x$	✓ S ✓ S ✓ S (3)
9.4	$\hat{A}_1 = \hat{C}$ (ext. $\angle$ of a cyclic quadrilateral / *buite* $\angle$ *van koordevier hoek*) $= 90° - 2x + x + x$ $\hat{A}_1 = 90°$ ED is a diameter of circle AED (line subtends $90° \angle$ / converse of $\angle$ in a semi circle) ED *is 'n middellyn van sirkel AED* (*lyn onderspan* $90° \angle$ *of* *omgekeerde van* $\angle$ *in halwe sirkel*)	✓ S ✓ R ✓ $\hat{A}_1 = 90°$ ✓ R (4)

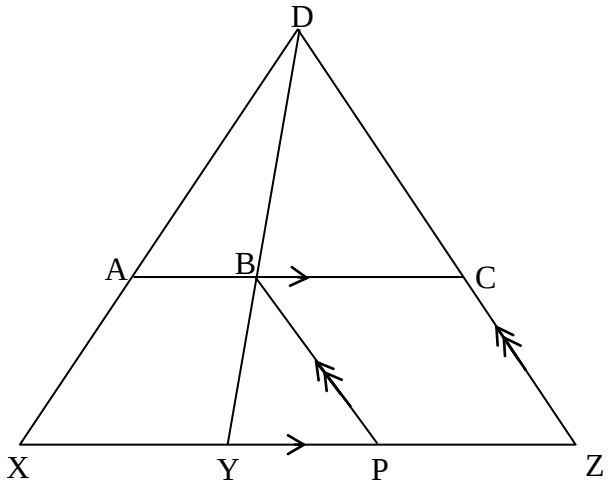
[14]

QUESTION / *VRAAG* 10



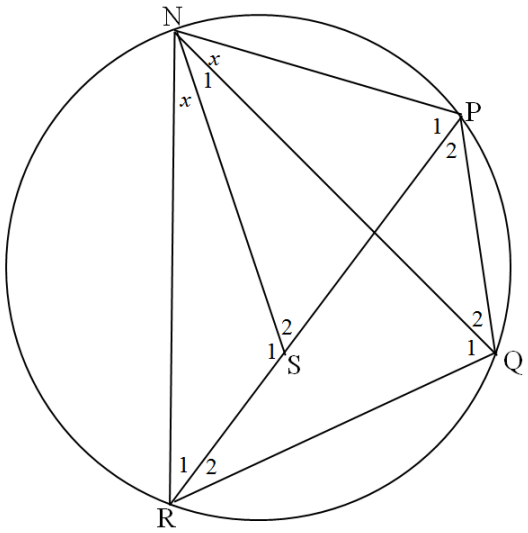
10.1	Construction: In Δ ADE, draw height h relative to base AD and height k relative to base AE. Join BE and DC to create Δ BDE and Δ CED. / *Konstruksie: In ΔADE, trek hoogte h relatief tot basis AD en die hoogte k relatief tot basis AE. Verbind BE en DC om ΔBDE en Δ CED te vorm.* Proof: / *Bewys:* $\frac{\text{Area } \Delta ADE}{\text{Area } \Delta BED} = \frac{\frac{1}{2}AD.h}{\frac{1}{2}BD.h} = \frac{AD}{DB}$ $\frac{\text{Area } \Delta ADE}{\text{Area } \Delta CED} = \frac{\frac{1}{2}AE.k}{\frac{1}{2}CE.k} = \frac{AE}{EC}$ but Area of ΔBED = Area of / *van* ΔCED [same base, same height / *dies. basis; dies. hoogte*] $\therefore \frac{\text{Area } \Delta ADE}{\text{Area } \Delta BED} = \frac{\text{Area } \Delta ADE}{\text{Area } \Delta CED}$ $\therefore \frac{AD}{DB} = \frac{AE}{EC}$	✓ construction / *konstruksie* ✓ S ✓ S ✓ S✓R ✓ S (6)

10.2



10.2	In $\Delta DXY: \frac{DA}{DX} = \frac{DB}{DY}$	[line $\parallel$ to one side of Δ *or* prop.theorem ; AB $\parallel$ XY / *lyn* $\parallel$ *een sy van* Δ *of ewer. stelling* ; AB $\parallel$ XY]	✓ S✓R
	In DYZ: $\frac{ZP}{ZY} = \frac{DB}{DY}$	[line $\parallel$ to one side of Δ *or* prop.theorem ; BC $\parallel$ YZ / *lyn* $\parallel$ *een sy van* Δ *of ewer. stelling* ; BC $\parallel$ YZ]	✓ S
	$\frac{DA}{DX} = \frac{ZP}{ZY}$		✓ S
	$ZP = BC$	[opp. sides of a parm / *teenoorst sye van* $\parallel^m$]	✓ S / R
	$\frac{BC}{YZ} = \frac{DA}{DX}$		(5) **[11]**

QUESTION / *VRAAG* 11



11.1	In ΔNSR and ΔNPQ $R\hat{N}S = P\hat{N}Q$ [given /*gegee*] $\hat{R}_1 = \hat{Q}_2$ [$\angle^s$ in the same segment / *omtr* $\angle^e$ *in dies sirkel segm*] $\hat{S}_1 = N\hat{P}Q$ [sum of $\angle^s$ in a Δ / *som* $\angle^e$ *van* Δ / $\angle^e$ *van* Δ] $\therefore \Delta NRS \|\|\| \Delta NQP$ [$\angle,\angle,\angle$]	✓ S ✓S / R ✓ R (3)
11.2	In ΔNQR and ΔNPS $R\hat{N}Q = P\hat{N}S$ [$R\hat{N}S = P\hat{N}Q$] $\hat{Q}_1 = \hat{P}_1$ [$\angle^s$ in the same segment / *omtr* $\angle^e$ *in dies sirkel segm*] $\hat{R} = \hat{S}_2$ [sum of $\angle^s$ in a Δ / *som* $\angle^e$ *van* Δ / $\angle^e$ *van* Δ] $\therefore \Delta NQR \|\|\| \Delta NPS$ [$\angle,\angle,\angle$]	✓ S ✓ S ✓ R (3)
11.3	$\frac{QR}{PS} = \frac{NQ}{NP}$ [$\Delta NQR \|\|\| \Delta NPS$] $QR.NP = PS.NQ$ $\frac{NR}{NQ} = \frac{SR}{PQ}$ [$\Delta NRS \|\|\| \Delta NQP$] $NR.PQ = NQ.SR$ $NR.PQ + QR.NP = NQ.SR + PS.NQ$ $= NQ(SR + PS)$ $\therefore NR.PQ + NR.QR = NQ.PR$	✓ S / R ✓ S / R ✓ S ✓ S (4) **[10]**

TOTAL / *TOTAAL* [150]