



Education

KwaZulu-Natal Department of Education
REPUBLIC OF SOUTH AFRICA

MATHEMATICS

SEPTEMBER PREPARATORY

2018

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MARKS: 150

TIME: 3 hours

N.B. This question paper consists of 8 pages and an information sheet.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 13 questions.
2. Answer **ALL** questions.
3. Clearly show **ALL** calculations, diagrams, graphs, et cetera that you have used in determining your answers.
4. Answers only will not necessarily be awarded full marks.
5. An approved scientific calculator (non-programmable and non-graphical) may be used, unless stated otherwise.
6. If necessary, answers should be rounded off to TWO decimal places, unless stated otherwise.
7. Diagrams are NOT necessarily drawn to scale.
8. Number the answers correctly according to the numbering system used in this question paper. Write neatly and legibly.

QUESTION 11.1 Solve for x :

1.1.1 $\frac{x}{2}(x-5)=0$ (2)

1.1.2 $3x^2 + 4x = 2$, correct to TWO decimal places. (4)

1.1.3 $\sqrt{2x+3} = x$ (4)

1.1.4 $9^x = 4.3^x$, correct to TWO decimal places. (5)

1.2 Solve the following equations simultaneously:

$$x = 2y \quad \text{and} \quad \frac{-4}{x} + \frac{y}{2} = 1\frac{1}{2}$$
 (6)

1.3 $2^{-x}(x+4) \leq 0$ (4)

[25]**QUESTION 2**

The first four terms of a quadratic sequence are 9 ; 19 ; 33 ; 51 ;...

2.1 Write down the next TWO terms of the quadratic sequence. (2)

2.2 Determine the n^{th} term of the sequence. (4)

2.3 Prove that all the terms of the quadratic sequence are odd. (3)

[9]**QUESTION 3** $3-t ; -t ; \sqrt{9-2t}$ are the first three terms of an arithmetic sequence.3.1 Determine the value of t . (4)3.2 If $t = -8$, then determine the number of terms in the sequence that will be positive. (3)**[7]**

QUESTION 4

4.1 Given the infinite geometric series $(x-3) + (x-3)^2 + (x-3)^3 + \dots$

4.1.1 Write down the value of the common ratio in terms of x . (1)

4.1.2 For which value(s) of x will the series converge? (3)

4.2 An arithmetic sequence and a geometric sequence have their first term as 3. The common difference of the arithmetic sequence is p and the common ratio of the geometric sequence is p . If the tenth term of the arithmetic sequence is equal to the sum to infinity of the geometric sequence, determine the value of p . (5)

[9]

QUESTION 5

Given $f(x) = \frac{x-3}{x+2}$

5.1 Show that $f(x) = 1 - \frac{5}{x+2}$ (1)

5.2 Write down the equations of the vertical and horizontal asymptotes of f . (2)

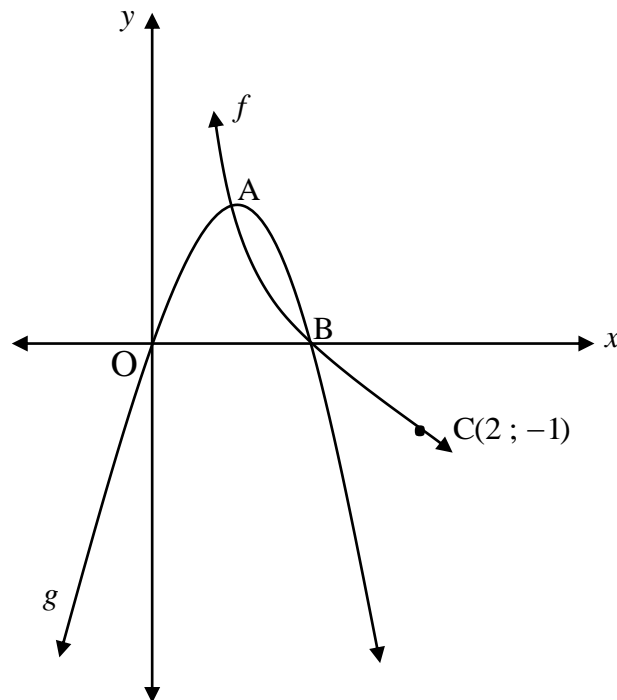
5.3 Determine the intercepts of the graph of f with the x -axes and y -axes. (2)

5.4 Write down the value of c if $y = x + c$ is a line of symmetry to the graph of f . (2)

[7]

QUESTION 6

$f(x) = \log_p x$ and $g(x) = ax^2 + bx$ are sketched below. A is the turning point of f and B is the common x -intercept of f and g . The point $C(2; -1)$ lies on the graph of f .



- 6.1 Calculate the value of p . (2)
- 6.2 Write down the co-ordinates of B. (1)
- 6.3 If $p = \frac{1}{2}$, determine the co-ordinates of A. (3)
- 6.4 Determine the values of a and b . (4)
- 6.5 Write down the equation of f^{-1} , inverse of f , in the form $y = \dots$ (2)
- 6.6 Determine the values of x for which $f(x) \geq -1$. (2)
- 6.7 Determine the values of x for which $f(x) \cdot g'(x) \leq 0$. (2)

[16]**QUESTION 7**

- 7.1 Consider the curve $y = -2x^3 + 3x^2 + 32x + 15$

Calculate the equation of the tangent to this curve at the point $(-2; -21)$. (5)

- 7.2 Determine the x -value of another point on this curve where the tangent calculated in question 7.1 intersects this curve again. (5)

[10]

QUESTION 8

- 8.1 A tractor costing R180 000 depreciates on the reducing balance method to R65 000 at the end of 8 years. Determine the rate at which the tractor is depreciating per annum. (3)
- 8.2 Tebogo buys a flat at the beach front for R850 000. She takes out a loan from the bank at an interest rate of 14,25 % per annum compounded monthly. Her first instalment will commence in one month after she has taken out the loan.
- 8.2.1 Calculate the monthly repayments over a period of 20 years. (4)
- 8.2.2 If the monthly repayment is increased by 20 % before the first payment is being made towards the loan, determine the number of payments that will now be made to settle the loan. (4)
- 8.2.3 Calculate the final payment to settle the loan in question 8.2.2. (4)

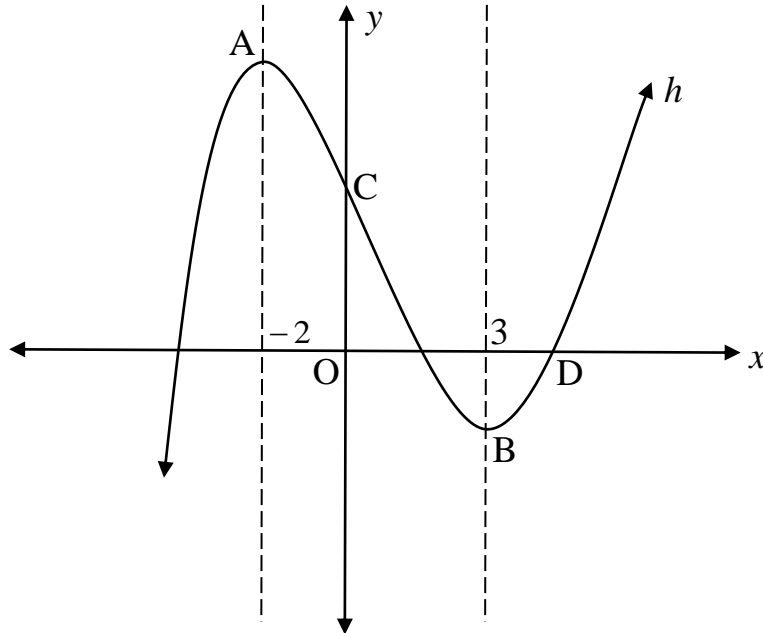
[15]**QUESTION 9**

- 9.1 Determine the derivative of $f(x) = -5x^2 + 3x$ from first principles. (5)
- 9.2 Calculate $g'(4)$ if $g(x) = \frac{1}{2\sqrt{x}}$ (4)
- 9.3 Determine $D_x[(2x - 3)^3]$ (4)

[13]

QUESTION 10

$h(x) = x^3 - \frac{3}{2}x^2 + cx + d$ is sketched below. A and B are the turning points of h at $x = -2$ and $x = 3$ respectively. C is the y -intercept of h . D is the point $(4; 0)$.



- 10.1 Show that $c = -18$ and $d = 32$. (5)
- 10.2 Calculate the co-ordinates of A. (2)
- 10.3 Determine the x -value of the point of inflection. (2)
- 10.4 Write down the interval for which h is concave up. (1)
- 10.5 If $g(x) = h(-x)$, write down the co-ordinates of the turning point that is the image of A. (2)
- 10.6 Determine the values of k for which $h(x) = k$ has 2 unequal negative real roots and one positive real root. (2)

[14]

QUESTION 11

The depth of water (in metres) left in the dam, t hours, after the sluice gate was opened to allow the flow of water to drain from the dam is given by the equation

$$D(t) = 28 - \frac{1}{9}t^2 - \frac{1}{27}t^3.$$

- 11.1 Calculate the average rate of change in the depth of the water after the first 2 hours. (4)
 11.2 Determine the rate at which the level of the water is decreasing after 16 hours. (4)
[8]

QUESTION 12

Study the table below and answer the questions that follow.

	Like Sport	Do not like Sport	Totals
Males	80	b	c
Females	a	90	d
Totals	200	150	350

- 12.1 Write down the values of a , b , c and d . (4)
 12.2 Is the event liking a sport independent of gender? Show all working. (4)
[8]

QUESTION 13

Consider the letters of the word "DEPENDENT". Determine, using all letters

- 13.1 the number of unique arrangements of the letters that can be formed? (3)
 13.2 the number of unique arrangements of letters that can be formed in 13.1 starting with the letter "N"? (3)
 13.3 the number of unique arrangements of letters that can be formed in 13.1 starting and ending with the letter "N"? (3)

[9]**Total Marks : 150**

INFORMATION SHEET: MATHEMATICS

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$A = P(1 + ni)$$

$$A = P(1 - ni)$$

$$A = P(1 - i)^n$$

$$A = P(1 + i)^n$$

$$T_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}; \quad r \neq 1$$

$$S_\infty = \frac{a}{1 - r}; \quad -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i}$$

$$P = \frac{x[1 - (1+i)^{-n}]}{i}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$M\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

$$y = mx + c$$

$$y - y_1 = m(x - x_1)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \tan \theta$$

$$(x-a)^2 + (y-b)^2 = r^2$$

$$\text{In } \triangle ABC: \quad \frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$a^2 = b^2 + c^2 - 2bc \cdot \cos A \quad \text{area } \triangle ABC = \frac{1}{2} ab \cdot \sin C$$

$$\sin(\alpha + \beta) = \sin \alpha \cdot \cos \beta + \cos \alpha \cdot \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cdot \cos \beta - \cos \alpha \cdot \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cdot \cos \beta - \sin \alpha \cdot \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cdot \cos \beta + \sin \alpha \cdot \sin \beta$$

$$\cos 2\alpha = \begin{cases} \cos^2 \alpha - \sin^2 \alpha \\ 1 - 2\sin^2 \alpha \\ 2\cos^2 \alpha - 1 \end{cases}$$

$$\sin 2\alpha = 2\sin \alpha \cdot \cos \alpha$$

$$\bar{x} = \frac{\sum f \cdot x}{n}$$

$$\sigma^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx$$

$$b = \frac{\sum (x - \bar{x})(y - \bar{y})}{\sum (x - \bar{x})^2}$$