



Education

KwaZulu-Natal Department of Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS

SEPTEMBER PREPARATORY 2018

MEMORANDUM

**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

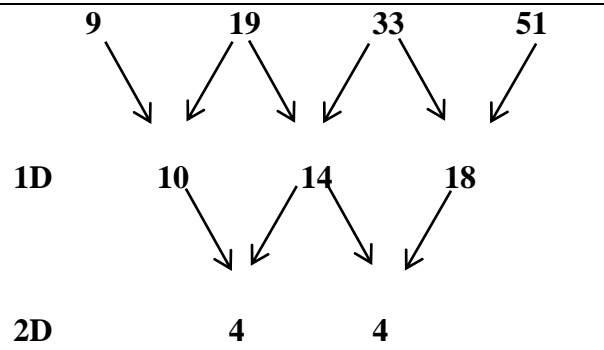
MARKS: 150

TIME: 3 hours

This memorandum consists of 14 pages.

<p>1.2</p>	<p>$x = 2y$ and $\frac{-4}{x} + \frac{y}{2} = 1\frac{1}{2}$</p> <p>$\frac{-4}{2y} + \frac{y}{2} = \frac{3}{2}$</p> <p>$-4 + y^2 = 3y$</p> <p>$y^2 - 3y - 4 = 0$</p> <p>$(y-4)(y+1) = 0$</p> <p>$y = 4$ or $y = -1$</p> <p>$x = 8$ or $x = -2$</p>	<p>A✓ substitution</p> <p>CA✓ simplification</p> <p>CA✓ standard form</p> <p>CA✓ factors</p> <p>CA✓ both y – values</p> <p>CA✓ both x – values</p>	<p>(6)</p>
<p>1.3</p>	<p>$2^{-x}(x+4) \leq 0$</p> <p>$2^{-x} > 0$ for all $x \in R$</p> <p>$\therefore x+4 \leq 0$</p> <p>$x \leq -4$</p>	<p>AA✓✓ $2^{-x} > 0$</p> <p>CA✓ $x+4 \leq 0$</p> <p>CA✓ answer</p> <p>OR</p> <p>If graphical Solution is used:</p> <p>2 Marks for sketches (AA)</p> <p>2 Marks for solution (CACA)</p>	<p>(4)</p>
			<p>[25]</p>

QUESTION 2

<p>2.1</p>	<p>73 ; 99</p>	<p>AA✓✓ answers</p>	<p>(2)</p>
<p>2.2</p>	 <p>1D</p> <p>2D</p> <p>$2a = 4$ $a = 2$</p> <p>$3a + b = 10$ $b = 4$</p> <p>$a + b + c = 9$ $c = 3$</p> <p>$T_n = 2n^2 + 4n + 3$</p>	<p>A✓ a value</p> <p>CA✓ b value</p> <p>CA✓ c value</p> <p>CA✓ answer</p>	<p>(4)</p>

	<p>OR</p> $T_n = T_1 + (n-1)d_1 + \frac{(n-1)(n-2)}{2}d_2$ $= 9 + (n-1)(10) + \frac{(n-1)(n-2)}{2}(4)$ $= 9 + 10n - 10 + 2n^2 - 6n + 4$ $= 2n^2 + 4n + 3$	<p>OR</p> <p>A✓ formula</p> <p>CA✓ substitution into correct formula</p> <p>CA✓ simplifying</p> <p>CA✓ answer</p>	(4)
2.3	$T_n = 2n^2 + 4n + 3$ $= 2(n^2 + 2n + 1) + 1$ <p>$2(n^2 + 2n + 1)$ is even for all $n \in \mathbb{N}$</p> <p>$\therefore 2(n^2 + 2n + 1) + 1$ is odd for all $n \in \mathbb{N}$</p> <p>OR</p> <p>For the first difference</p> $T_n = 4n + 6 = 2(2n + 3)$ <p>An even number of the first difference is always added to first term of the quadratic sequence to get an odd number. This process continues to produce all odd numbers of the sequence.</p>	<p>CA✓ rewriting n^{th} term</p> <p>A✓ reasoning</p> <p>A✓ reasoning</p> <p>OR</p> <p>CA✓ for nth term of first difference</p> <p>A✓ reasoning</p> <p>A✓ reasoning</p> <p>NB. If a candidate presents the following argument: The first term (9) is odd. To get next term, an even number is always added. This will give an odd number all the time.</p> <p>Award 1/3</p>	(3)
			[9]

QUESTION 3

3.1	$3-t; -t; \sqrt{9-2t}$ $-t-(3-t) = \sqrt{9-2t} - (-t)$ $-t-3+t = \sqrt{9-2t} + t$ $-3-t = \sqrt{9-2t}$ $9+6t+t^2 = 9-2t$ $t^2+8t=0$ $t(t+8)=0$ $t=0$ or $t=-8$ <i>n/a</i>	A✓ equating differences CA✓ standard form of equation CA✓ factors CA✓ answers with rejection	(4)
3.2	Pattern is 11; 8 ; 5 ; 2 ; -1 ; ... ∴ 4 terms are positive. OR 11; 8 ; 5 $T_n = -3n + 14 > 0$ $n < \frac{14}{3}$ <i>i.e.</i> $n < 4\frac{2}{3}$ 4 terms are positive.	<div style="border: 1px solid black; padding: 5px; display: inline-block; margin: 10px;"> Answer only full marks </div> AA✓✓ all 5 terms listed CA✓ answer CA✓ n^{th} term > 0 CA✓ $n < 4\frac{2}{3}$ CA✓ conclusion	(3)
			[7]

QUESTION 4

4.1.1	$r = (x-3)$	A✓ answer	(1)
4.1.2	$-1 < r < 1$ $-1 < x-3 < 1$ $2 < x < 4$	A✓ condition CA✓ substitution of common ratio CA✓ answer	(3)

<p>4.2</p>	<p>3 ; 3 + p ; 3 + 2p ; ... and 3 ; 3p ; 3p² ;</p> <p>$T_{10} = 3 + 9p$</p> <p>$S_{\infty} = \frac{3}{1-p}$</p> <p>$T_{10} = 3 + 9p = \frac{3}{1-p}$</p> <p>$(3 + 9p)(1 - p) = 3$</p> <p>$3 + 6p - 9p^2 = 3$</p> <p>$9p^2 - 6p = 0$</p> <p>$3p(3p - 2) = 0$</p> <p>$p = 0$ or $p = \frac{2}{3}$</p> <p>n/a</p>	<p>A✓ 3 + 9p</p> <p>A✓ $\frac{3}{1-p}$</p> <p>CA✓ equating</p> <p>CA✓ standard form</p> <p>CA✓ p – values and rejecting</p>	<p>(5)</p>
			<p>[9]</p>

QUESTION 5

<p>5.1</p>	<p>$f(x) = \frac{x+2}{x+2} - \frac{5}{x+2}$</p> <p>$= 1 - \frac{5}{x+2}$</p>	<p>A✓ writing numerator as x + 2 – 5</p>	<p>(1)</p>
<p>5.2</p>	<p>x = -2 and y = 1</p>	<p>A✓ x = -2 A✓ y = 1</p>	<p>(2)</p>
<p>5.3</p>	<p>y – intercept: $(0; -\frac{3}{2})$</p> <p>x – intercept: (3; 0)</p>	<p>A✓ y – intercept</p> <p>A✓ x – intercept</p> <p>(co-ordinate form not needed)</p>	<p>(2)</p>
<p>5.4</p>	<p>$y = x + c$</p> <p>$1 = -2 + c \therefore c = 3$</p> <p>OR</p> <p>$f(x) = \frac{x-3}{x+2} = \frac{x+2-5}{x+2} = \frac{-5}{x+2} + 1$</p> <p>$y = x + 2 + 1 = x + 3$</p> <p>$c = 3$</p>	<p>CA✓ substitution of the point (-2 ; 1)</p> <p>CA✓ answer</p> <p>OR</p> <p>CA✓ y = x + 3 (m must be 1)</p> <p>CA✓ answer</p>	<p>(2)</p> <p>(2)</p>
			<p>[7]</p>

QUESTION 6

6.1	$f(x) = \log_p x$ $-1 = \log_p 2$ $p^{-1} = 2$ $p = \frac{1}{2}$	A✓ substitution of the point (2 ; - 1) A✓ answer	(2)
6.2	$B(1 ; 0)$	A✓ answer	(1)
6.3	At A the x – co-ordinate is the same as the axis of symmetry value of the graph of g . $x = \frac{1}{2}$ $\therefore y = \log_{\frac{1}{2}} \frac{1}{2}$ $= 1$ $A\left(\frac{1}{2}; 1\right)$	CA✓ x – value CA✓ substitution CA✓ answer	(3)
6.4	$y = a(x - 0)(x - 1)$ $1 = a\left(\frac{1}{2} - 0\right)\left(\frac{1}{2} - 1\right)$ $1 = -\frac{1}{4}a \quad \therefore a = -4$ $y = -4x(x - 1)$ $y = -4x^2 + 4x$ $b = 4$ OR	CA✓ substitution of x intercepts and TP CA✓ a – value ($a < 0$) CA✓ substitution into equation CA✓ b – value OR	(4)

	$y = a(x + p)^2 + q$ $y = a\left(x - \frac{1}{2}\right)^2 + 1$ <p>$B(1; 0)$:</p> $0 = a\left(-\frac{1}{2}\right)^2 + 1$ $-1 = \frac{1}{4}a$ $a = -4$ $y = -4\left(x - \frac{1}{2}\right)^2 + 1$ $= -4\left(x^2 - x + \frac{1}{4}\right) + 1$ $= -4x^2 + 4x - 1 + 1$ $= -4x^2 + 4x$ <p>$\therefore b = 4$</p> <p>OR</p> <p>$B(1;0) : 0 = a + b \rightarrow (1)$</p> <p>$A\left(\frac{1}{2}; 1\right) : 1 = \frac{1}{4}a + \frac{1}{2}b \rightarrow (2)$</p> <p>$(2) : 4 = a + 2b \rightarrow (3)$</p> <p>Substituting $a = -b$ into (3)</p> $4 = -b + 2b$ <p>$\therefore b = 4$</p> $a = -4$	<p>CA✓ substitution of pt. B and TP.</p> <p>CA✓ a – value</p> <p>CA✓ equation of parabola CA✓ b – value</p> <p>OR</p> <p>CA✓ subst. $B(1; 0)$ to form eq. (1)</p> <p>CA✓ subst. $A\left(\frac{1}{2}; 1\right)$ to form eq. (2)</p> <p>CA✓ b – value CA✓ a – value</p>	<p>(4)</p> <p>(4)</p>
6.5	$y = \left(\frac{1}{2}\right)^x \quad \text{or} \quad y = 2^{-x}$	AA✓✓ answer	(2)
6.6	$(0; 2]$	AA✓✓ answer (penalize 1 for incorrect notation)	(2)
6.7	$\frac{1}{2} \leq x \leq 1$	CACA✓✓ answer (penalize 1 for incorrect notation)	(2)
			[16]

QUESTION 7

7.1	$y = -2x^3 + 3x^2 + 32x + 15$ $\frac{dy}{dx} = -6x^2 + 6x + 32$ $m = -6(-2)^2 + 6(-2) + 32 = -4$ $y = mc + c$ $-21 = -4(-2) + c$ $c = -29$ $y = -4x - 29$	A✓ derivative CA✓ substitution of $x = -2$ into derivative and equating to gradient CA✓ substituting $m = -4$ and given point CA✓ c – value CA✓ answer	(5)
7.2	$-2x^3 + 3x^2 + 32x + 15 = -4x - 29$ $-2x^3 + 3x^2 + 36x + 44 = 0$ $2x^2 - 3x^2 - 36x - 44 = 0$ $(x + 2)(x + 2)(2x - 11) = 0$ $x = -2 \quad \text{or} \quad x = \frac{11}{2} = 5.5 = 5\frac{1}{2}$ $x = \frac{11}{2} = 5.5 = 5\frac{1}{2}$	CA✓ equating CA✓ standard form CA✓ factors CA✓ x – values CA✓ choosing answer	(5)
			[10]

QUESTION 8

8.1	$A = P(1 - i)^n$ $65000 = 180\,000(1 - i)^8$ $\frac{65\,000}{180\,000} = (1 - i)^8$ $1 - i = \sqrt[8]{\frac{65\,000}{180\,000}}$ $i = 1 - \sqrt[8]{\frac{65\,000}{180\,000}}$ $i = 0,1195491715$ <p>Therefore the interest rate is 11,95 % p.a.</p>	<p>A✓ substitution into correct formula</p> <p>A✓ i – value</p> <p>A✓ answer</p>	(3)
8.2.1	$P_v = \frac{x[1 - (1 + i)^{-n}]}{i}$ $850\,000 = \frac{x \left[1 - \left(1 + \frac{0,1425}{12} \right)^{-240} \right]}{\frac{0,1425}{12}}$ $x = R10\,724,61$	<p>A✓ formula</p> <p>A✓ substitution of P value</p> <p>A✓ substitution of i and n values</p> <p>CA✓ answer</p> <p>N.B. Substituting i and n in the future value formula – Award 1/4</p>	(4)
8.2.2	$120\% \text{ of } \frac{10724,61}{1} = R12869,53$ $P_v = \frac{x[1 - (1 + i)^{-n}]}{i}$ $850\,000 = \frac{12869,53 \left[1 - \left(1 + \frac{0,1425}{12} \right)^{-n} \right]}{\frac{0,1425}{12}}$ $\left(1 + \frac{0,1425}{12} \right)^{-n} = 0,2156861983$ $-n = \log_{\left(1 + \frac{0,1425}{12} \right)} 0,2156861983$ $-n = -129,938569$ $\therefore n = 129,938569$ $n = 130 \text{ payments}$	<p>CA✓ $x = 12869,53$</p> <p>CA✓ substitution into correct formula</p> <p>CA✓ use of logs</p> <p>CA✓ answer</p>	(4)

<p>8.2.3</p>	<p>Balance on loan</p> $P_v = \frac{12869,53 \left[1 - \left(1 + \frac{0,1425}{12} \right)^{-0,938569} \right]}{\frac{0,1425}{12}}$ <p>= R11 941,51</p> <p>Final Instalment = $11941,51 \left(1 + \frac{0,1425}{12} \right) = 12083,32$</p> <p>OR</p> <p>Balance on loan = A - F</p> $= 850000 \left(1 + \frac{0,1425}{12} \right)^{129} - \frac{12869,53 \left[\left(1 + \frac{0,1425}{12} \right)^{129} - 1 \right]}{\frac{0,1425}{12}}$ <p>= R11 941,51</p> <p>Final Instalment = $11941,51 \left(1 + \frac{0,1425}{12} \right) = 12083,32$</p>	<p>CA✓ n – value</p> <p>CA✓ substitution into present value formula</p> <p>CA✓ R11 941,51</p> <p>CA✓ R12083,32</p> <p>OR</p> <p>CA✓ n – value</p> <p>CA✓ substitution into formulae</p> <p>CA✓ R11 941,51</p> <p>CA✓ R12083,32</p>	<p>(4)</p> <p>(4)</p> <p>[15]</p>
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QUESTION 9 (penalize 1 mark once for incorrect notation in this question)

<p>9.1</p>	$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-5(x+h)^2 + 3(x+h) - (-5x^2 + 3x)}{h}$ $= \lim_{h \rightarrow 0} \frac{-5x^2 - 10xh - 5h^2 + 3x + 3h + 5x^2 - 3x}{h}$ $= \lim_{h \rightarrow 0} \frac{h(-10x - 5h + 3)}{h}$ <p>= -10x + 3</p>	<p>A✓ formula</p> <p>A✓ substitution</p> <p>CA✓ simplification of numerator</p> <p>CA✓ factorization</p> <p>CA✓ answer</p>	<p>(5)</p>
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<p>9.2</p>	$g(x) = \frac{1}{2\sqrt{x}} = \frac{1}{2}x^{-\frac{1}{2}}$ $g'(x) = -\frac{1}{4}x^{-\frac{3}{2}}$ $g'(4) = -\frac{1}{4}(4)^{-\frac{3}{2}} = -\frac{1}{4}(2^2)^{-\frac{3}{2}}$ $= -\frac{1}{4} \cdot \frac{1}{8} = -\frac{1}{32}$	<p>A✓rewriting in exponential form</p> <p>CA✓derivative</p> <p>CA✓substituting 4 into derivative</p> <p>CA✓answer</p>	<p>(4)</p>
<p>9.3</p>	$D_x[(2x-3)^3]$ $= D_x[8x^3 - 36x^2 + 54x - 27]$ $= 24x^2 - 72x + 54$	<p>A✓cubing the binomial</p> <p>CACACA✓✓✓each answer</p>	<p>(4)</p>
<p></p>	<p></p>	<p></p>	<p>[13]</p>

QUESTION 10

<p>10.1</p>	$h(x) = x^3 - \frac{3}{2}x^2 + cx + d$ $h'(x) = 3x^2 - 3x + c$ $h'(3) = 3(3)^2 - 3(3) + c = 0$ $27 - 9 + c = 0$ $c = -18$ $h(x) = x^3 - \frac{3}{2}x^2 - 18x + d$ $h(4) = (4)^3 - \frac{3}{2}(4)^2 - 18(4) + d = 0$ $64 - 24 - 72 + d = 0$ $d = 32$ <p>OR</p> $h'(x) = 3x^2 - 3x + c$ $h'(x) = 3(x+2)(x-3) = 3x^2 - 3x - 18$ $c = -18$ $h(x) = x^3 - \frac{3}{2}x^2 - 18x + d$ $h(4) = 64 - 24 - 72 + d = 0$ $d = 32$	<p>A✓derivative</p> <p>A✓subst. 3 or - 2 into derivative and equating to 0</p> <p>A✓simplifying</p> <p>A✓subst. 4 into h and equating to 0</p> <p>A✓simplifying</p> <p>OR</p> <p>A✓derivative</p> <p>A✓derivative using stationary values</p> <p>A✓simplifying</p> <p>A✓equating coefficients of polynomials to get c - value</p> <p>A✓substituting x = 4 onto equation to get d - value</p>	<p>(5)</p> <p>(5)</p>
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10.2	$h(x) = x^3 - \frac{3}{2}x^2 - 18x + 32$ $h(-2) = (-2)^3 - \frac{3}{2}(-2)^2 - 18(-2) + 32 = 54$ $A(-2 ; 54)$	A✓ subst. $x = -2$ into h A✓ y - value	(2)
10.3	$x = \frac{-2+3}{2}$ $x = \frac{1}{2}$ <p>OR</p> $h(x) = x^3 - \frac{3}{2}x^2 - 18x + 32$ $h'(x) = 3x^2 - 3x - 18$ $h''(x) = 6x - 3 = 0$ $x = \frac{1}{2}$	A✓ $x = \frac{-2+3}{2}$ CA✓ answer OR A✓ second derivative equal to 0 CA✓ x - value	(2)
10.4	$x > \frac{1}{2}$	CA✓ answer	(1)
10.5	(2 ; 54)	A $x=2$ CA $y=54$ ✓✓ answer	(2)
10.6	$32 < k < 54$	CACA ✓✓ answer	(2)
			[14]

QUESTION 11

11.1	$D \left[28 - \frac{1}{9}t^2 - \frac{1}{27}t^3 \right].$ $D(2) = 28 - \frac{1}{9}(2)^2 - \frac{1}{27}(2)^3 = \frac{736}{27} = 27\frac{7}{27} = 27,26$ $\text{Average Rate of change} = \frac{27,26 - 28}{2 - 0} = -\frac{10}{27} = -0,37$	A✓ subst. $t = 2$ A✓ 27,26 CA✓ subst. into average rate of change CA✓ answer	(4)
11.2	$D = 28 - \frac{1}{9}t^2 - \frac{1}{27}t^3.$ $D'(t) = -\frac{2}{9}t - \frac{1}{9}t^2$ $D'(16) = -\frac{2}{9}(16) - \frac{1}{9}(16)^2$ $= -32m/h$ <p>The water level is decreasing at 32 m/h.</p>	A✓ A✓ derivative CA✓ subst. $t = 16$ CA✓ $-32m/h$	(4)
			[8]

QUESTION 12

12.1	$a = 120 ; b = 60 ; c = 140 ; d = 210$	A✓a – value and A✓b – value A✓c – value and A✓d – value	(4)
12.2	<p>$P(\text{Male}) = \frac{140}{350}$</p> <p>$P(\text{liking sport}) = \frac{200}{350}$</p> <p>$P(\text{Male and liking sport}) = \frac{80}{350} = \frac{8}{35}$</p> <p>$P(\text{Male}) \times P(\text{liking sport})$ $= \frac{140}{350} \times \frac{200}{350} = \frac{8}{35}$</p> <p>$P(\text{Male liking sport}) = P(\text{Male}) \times P(\text{liking sport})$ \therefore The events are independent.</p> <p>OR</p> <p>$P(\text{Female}) = \frac{210}{350}$</p> <p>$P(\text{liking sport}) = \frac{200}{350}$</p> <p>$P(\text{Female and liking sport}) = \frac{120}{350} = \frac{12}{35}$</p> <p>$P(\text{Female}) \times P(\text{liking sport})$ $= \frac{210}{350} \times \frac{200}{350} = \frac{12}{35}$</p> <p>$P(\text{Female liking sport}) = P(\text{Female}) \times P(\text{liking sport})$ \therefore The events are independent.</p>	<p>CA✓ $P(\text{Male}) = \frac{140}{350}$</p> <p>CA✓ $P(\text{Male and liking sport}) = \frac{8}{35}$</p> <p>$P(\text{Male}) \times P(\text{liking sport})$ CA✓ $= \frac{140}{350} \times \frac{200}{350} = \frac{8}{35}$</p> <p>CA✓ conclusion</p> <p>OR</p> <p>CA✓ $P(\text{Female}) = \frac{140}{350}$</p> <p>CA✓ $P(\text{Female and liking sport}) = \frac{12}{35}$</p> <p>$P(\text{Female}) \times P(\text{liking sport})$ CA✓ $= \frac{210}{350} \times \frac{200}{350} = \frac{12}{35}$</p> <p>CA✓ conclusion</p>	<p>(4)</p> <p>(4)</p>
			[8]

QUESTION 13

13.1	<p>There are 9 letters: 3 Es, 2Ds and 2Ns. The number of different words are</p> $= \frac{9!}{3! \times 2! \times 2!}$ $= 15120$	<p>A✓ 9! A✓ $3! \times 2! \times 2!$ A✓ answer</p>	(3)
13.2	<p>If we take one of the letters for the first letter, there are seven letters remaining, of which there are 3Es and 2 Ds.</p> <p>Hence the number of words</p> $= \frac{1 \cdot 8!}{3! \times 2!}$ $= 3360$	<p>A✓ 8! A✓ $3! \times 2!$ A✓ answer</p>	(3)
13.3	<p>If both Ns are used for the first and last , there are 7 letters remaining of which there are 3Es and 2Ds. Hence the number of word</p> $= \frac{1 \cdot 7! \cdot 1}{3! \times 2!}$ $= 420$	<p>A✓ 7! A✓ $3! \times 2!$ A✓ answer</p>	(3)
			[9]

Total Marks : 150