education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

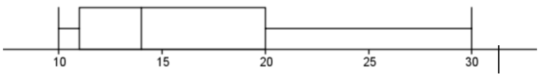
September 2018

MARKING GUIDELINES

MARKS: 150

This marking guideline consist of 20 pages

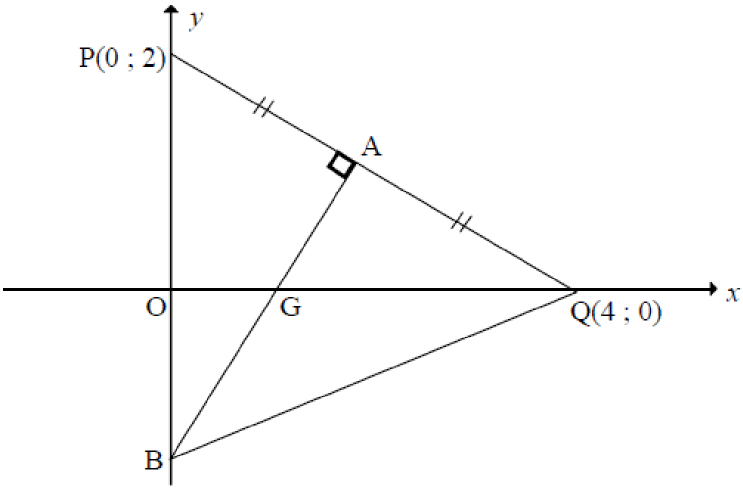
Question 1

1.1	IQR = $Q_3 - Q_1$ $= 20 - 11$ $= 9$	✓ $Q_3 - Q_1$ or $20 - 11$ ✓ 9 (2)
1.2		✓ min and max ✓median ✓upper and lower quartile (3)
1.3	Skewed to right or positively skewed	✓ answer (1)
		[6]

Question 2

2.1.1	By using a calculator : a = 29,22 (29.21542…) b = 0,89 (0,886530…) $\therefore$ equation of line of least squares is $y = 29.22 + 0.89x$	✓ a ✓ b ✓ equation (3)
2.1.2	y = 29.22 + (0.89)(32) = 57,7 OR 57,58 Therefore the employee who undergoes 32 hours of training should produce about 58 units.	✓substitution ✓answer as integer (2)
2.1.3	Moderate OR not very strong Because the value of r = 0.66	✓moderate ✓ reason (2)
2.2.1	$\bar{x} = \frac{45+70+44+56+60+48+75+60+63+38}{10} = \frac{559}{10}$ $= 55.9$	✓sum ✓answer (2)
2.2.2	SD = 11.36 $\bar{x} + SD$ $= 55.9 + 11.36$ $= 67.26$ $\therefore 2\,\text{employees}$	✓✓ SD ✓67.26 ✓2 employees (4)
		[13]

Question 3



3.1	$m_{PQ} = \frac{2-0}{0-4}$ $= -\frac{1}{2}$	✓ answer (1)
3.2	$A\left(\frac{0+4}{2}; \frac{2+0}{2}\right)$ $A(2\ ;1)$	✓ x_A ✓ y_A (2)
3.3	$m_{AB}.m_{PQ} = -1$ $m_{AB} \cdot \left(\frac{-1}{2}\right) = -1$ $m_{AB} = 2$ Equation of AB is $y = 2x + c$ $\therefore 1 = 2(2) + c$ $c = -3$ Equation of AB is $y = 2x - 3$ **OR**	✓ $m_{AB} = 2$ ✓ substitution of (2;1) and m ✓ equation of AB (3) ✓ $m_{AB} = 2$

	$m_{AB}.m_{PQ} = -1$ $m_{AB}.\left(\frac{-1}{2}\right) = -1$ $m_{AB} = 2$ $y-1 = 2(x-2)$ $y-1 = 2x-4$ Equation of AB is $y = 2x-3$	✓ Substitution of (2;1) ✓ equation of AB (3)
3.4	B is the point (0 ; –3) $\text{BQ} = \sqrt{(0-4)^2 + (-3-0)^2}$ $\text{BQ} = 5$ **OR** $\text{BQ}^2 = 4^2 + 3^2$ (Pyth) $\text{BQ} = 5$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)
3.5	If PBQR is a rhombus then A is the midpoint of BR. Let the coordinates of R be (x ; y) $\frac{x+0}{2} = 2$ AND $\frac{y-3}{2} = 1$ $x = 4$ $y = 5$ R(4 ; 5) **OR** RQ ǁ PB so $x_R = 4$ RQ = PB = 5, so $y_R = 5$ ∴R(4 ; 5)	✓✓ x coordinate ✓✓ y coordinate (4) ✓✓ x coordinate ✓✓ y coordinate (4)

3.6	$\tan A\hat{G}Q = 2$ $\therefore A\hat{G}Q = 63{,}43^\circ$ $m_{BQ} = \frac{-3-0}{0-4}$ $m_{BQ} = \frac{3}{4}$ $\therefore \tan\beta = \frac{3}{4}$ $\therefore \beta = 36{,}87°$ $\therefore G\hat{Q}B = 36{,}87^\circ$ (vertically opp angles) $63{,}43^\circ = 36.87^\circ + A\hat{B}Q$ (ext∠ of Δ) $\therefore A\hat{B}Q = 26{,}56^\circ$ **OR** $\tan A\hat{G}Q = 2$ $\therefore A\hat{G}Q = 63{,}43^\circ$ $\therefore B\hat{G}Q = 180^\circ - 63.43^\circ$ (∠s on straight line) $= 116.57^\circ$ $m_{BQ} = \frac{-3-0}{0-4}$ $m_{BQ} = \frac{3}{4}$ $\therefore \tan\beta = \frac{3}{4}$ $\therefore \beta = 36.87°$ $180 - 116{,}57^\circ - 36.87^\circ = A\hat{B}Q$ (∠ in Δ) $\therefore A\hat{B}Q = 26{,}56^\circ$	✓ 63.43° ✓ m_{BQ} ✓ 36,87° ✓exterior angle of triangle ✓answer (5) ✓ 116.57° ✓ m_{BQ} ✓ 36.87° ✓ angles in triangle ✓ answer (5)
3.7.1	$P\hat{A}B = 90^\circ$ $\therefore$ PB is the diameter (converse : ∠at center = 2 × ∠at circum)	✓ BQ is diameter

	$\text{Midpoint PB}\left(0\ ;\ -\frac{1}{2}\right)$ $x^2+\left(y+\frac{1}{2}\right)^2=r^2$ subst A(2 ; 1) $\therefore (2)^2+\left(1+\frac{1}{2}\right)^2=r^2$ $r=\frac{5}{2}$ $x^2+\left(y+\frac{1}{2}\right)^2=\frac{25}{4}$ **OR** subst B(0 ; −3) $2^2+\left(-3+\frac{1}{2}\right)^2=r^2$ $r^2=\frac{25}{4}$ $x^2+\left(y+\frac{1}{2}\right)^2=\frac{25}{4}$ subst B(0 ; 2) $\left(2+\frac{1}{2}\right)^2=r^2$ $r^2=\frac{25}{4}$ $x^2+\left(y+\frac{1}{2}\right)^2=\frac{25}{4}$	✓✓midpoint ✓substitution ✓ $r=\frac{5}{2}$ ✓equation LHS ✓equation RHS (6)
3.7.2	$y=2$ ($r\perp$ tangent)	✓ ✓ $y=2$ (2)
		[25]

Question 4

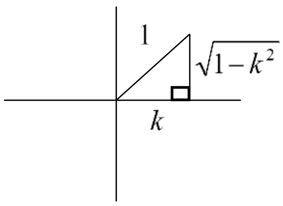
4.1.1	$x^2+y^2+8x+4y-38=0$ $x^2+8x+16+y^2+4y+4=38+16+4$ $(x+4)^2+(y+2)^2=58$ B(−4 ; −2)	✓ $(x+4)^2+(y+2)^2$ ✓ $r^2=58$ ✓ x_B ✓ y_B (4)

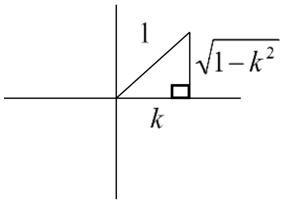
	OR $x_B = \frac{8}{-2} = -4$ $y_B = \frac{4}{-2} = -2$ $B(-4; -2)$	
4.1.2	$\text{radius} = \sqrt{58}$ OR $r = \sqrt{a^2 + b^2 - c}$ $r = \sqrt{16 + 4 + 38}$ $r = \sqrt{58}$	✓ radius (1)
4.2.1	Centre of second circle is (4 ; 6) Distance between of AB: $\sqrt{(4+4)^2 + (6+2)^2}$ $= \sqrt{128}$ OR 11.31	✓ centre ✓substitution ✓answer (3)
4.2.2	Sum of radii = $\sqrt{58} + \sqrt{26}$ = 12,71 Distance between centres is 11,31. sum of the radii > distance between the centres ∴ the circles must overlap and hence the circles must intersect.	✓✓ sum of radii ✓ conclusion (3)
4.3	$AB \perp CD$ (line from centre ⊥ chord) $m_{AB} = \frac{6+2}{4+4}$ $= \frac{8}{8}$ $= 1$ $\therefore m_{CD} = -1$ **OR**	✓S/R ✓subst ✓answer (3) ✓ S/R ✓subst

	AB ⊥ CD (line from centre ⊥ chord) $m_{AB} = \frac{-2-6}{-4-4}$ $= \frac{-8}{-8}$ $= 1$ $\therefore m_{CD} = -1$	✓answer (3)
		[14]

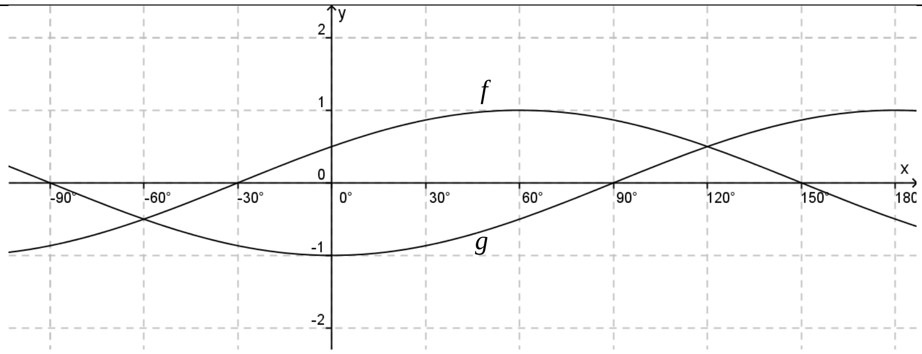
Question 5

5.1.1	$0{,}76604 \approx 0{,}77$	✓✓answer (2)
5.1.2	$\frac{1-\tan^2\theta}{1-\tan^2\theta}$ $= \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}}$ OR $\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$ $= \frac{\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$ $= \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta} \times \frac{\cos^2\theta}{\cos^2\theta+\sin^2\theta}$ $= \cos^2\theta - \sin^2\theta$ $= 2\cos^2\theta - 1$ $= \cos 2\theta$	✓ $\frac{\sin^2\theta}{\cos^2\theta}$ ✓ simplify ✓ $\cos^2\theta+\sin^2\theta = 1$ ✓ $\cos^2\theta - \sin^2\theta$ ✓ $2\cos^2\theta - 1$ (5)
5.1.3	$\frac{1-\tan^2\theta}{1-\tan^2\theta} = \frac{1}{2}$ $\therefore \cos 2\theta = \frac{1}{2}$ $\therefore ref\ \angle = 60^\circ$ $2\theta = 60^\circ + 360.k;\ k \in Z$ *or* $2\theta = 300^\circ + 360k;\ k \in Z$ $\theta = 30^\circ + 180k;\ k \in Z$ $\theta = 150^\circ + 180k,\ k \in Z$	✓ $\cos 2\theta = \frac{1}{2}$ ✓ 60° ✓ 300° ✓ 30° and 150° ✓ $+180k;\ k \in Z$ (5)

	OR $\frac{1-\tan^2\theta}{1-\tan^2\theta}=\frac{1}{2}$ $\therefore \cos 2\theta = \frac{1}{2}$ $\therefore ref\ \angle = 60^\circ$ $2\theta = \pm 60^\circ + 360.k;\ \ k \in Z$ $\theta = \pm 30^\circ + 180k.\ \ k \in Z$	✓ $\cos 2\theta = \frac{1}{2}$ ✓ 60° ✓ -60° ✓ 30° and -30° ✓ $+180k;\ k \in Z$ (5)
5.2.1	$\sin 245°$ $= \sin(180^\circ + 65^\circ)$ $= -\sin 65^\circ$ $= -\cos 25^\circ$ $= -k$	✓ $-\sin 65^\circ$ ✓ $-\cos 25^\circ$ ✓ $-k$ (3)
5.2.2	$\sin 25°$ $= \sqrt{1-k^2}$  OR $\sin^2 25° + \cos^2 25° = 1$ $\sin^2 25° = 1-k^2$ $\sin 25° = \sqrt{1-k}$	✓sketch ✓answer (2)
5.2.3	$\cos 50°$ $= 2\cos^2 25^\circ - 1$ $= 2k^2 - 1$ **OR** $\cos^2 25° - \sin^2 25°$ $= (k)^2 - \left(\sqrt{1-k^2}\right)^2$ $= k^2 - 1 + k^2$ $= 2k^2 - 1$ **OR** $1 - \sin^2 25°$	✓identity ✓answer (2) ✓identity ✓answer (2) ✓identity ✓answer (2)

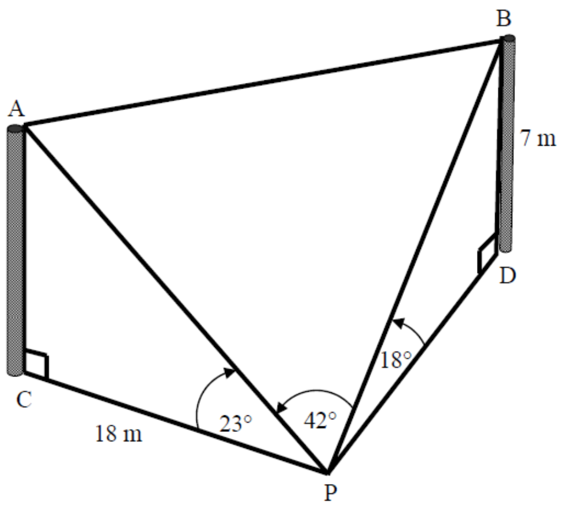
	$1-2(\sqrt{1-k^2})^2$ $=1-2(1-k^2)$ $=1-2+2k^2$ $=2k^2-1$	
5.2.3	$\sin 25°$ $=\sqrt{1-k^2}$ 	✓Sketch ✓answer (2)
5.3.1	$\sqrt{3}\cos\beta+\sin\beta$ $=2.\frac{\sqrt{3}}{2}.\cos\beta+\frac{2}{2}.\sin\beta$ $=2\left(\frac{\sqrt{3}}{2}.\cos\beta+\frac{1}{2}.\sin\beta\right)$ $=2(\sin 60^\circ.\cos\beta+\cos 60^\circ.\sin\beta)$ $=2\sin(60^\circ+\beta)$	✓ $\times\frac{2}{2}$ ✓ $2\left(\frac{\sqrt{3}}{2}.\cos\beta+\frac{1}{2}.\sin\beta\right)$ ✓$\sin 60^\circ$ ✓$\cos 60^\circ$ ✓ $2\sin(60^\circ+\beta)$ (5)
5.3.2	Max - value = 2 – 5 = – 3	✓✓answer (2)
		[26]

<u>Question 6</u>

6.1		$f(x)$ ✓shape ✓x-intercepts ✓y-intercepts ✓turning point (4)
6.2	$g(2x)=-\cos 2x$ $\therefore$ period $=180^\circ$	✓ $g(2x)=-\cos 2x$ ✓answer (2)

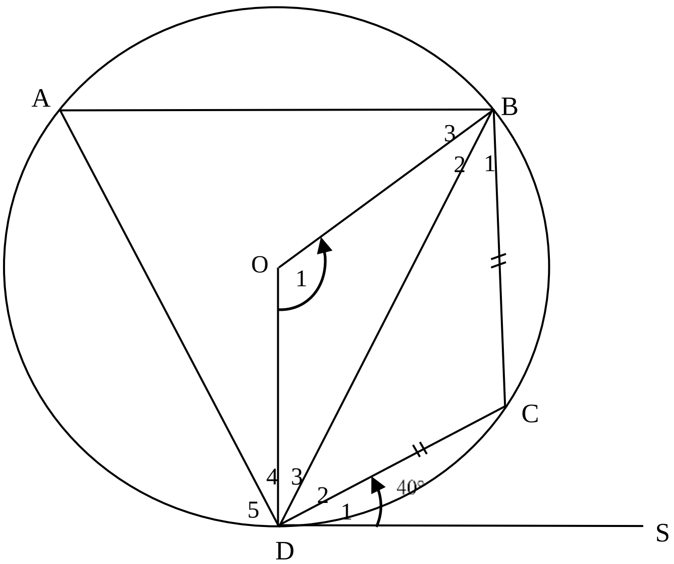
6.3	$0^\circ < x < 60^\circ$ **OR** $x \in (0^\circ;60^\circ)$	✓notation ✓end points (2)
		[8]

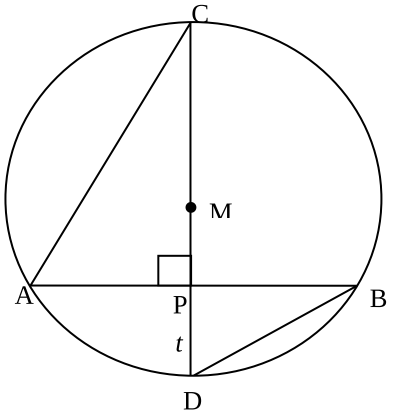
Question 7

		
7.1	$\frac{18}{PA} = \cos 23^\circ$ $PA = \frac{18}{\cos 23^\circ}$ $PA = 19.55m$ **OR** $\frac{AP}{\sin C} = \frac{CP}{\sin A}$ $\frac{AP}{\sin 90^\circ} = \frac{18}{\sin 67^\circ}$ $AP = \frac{18\sin 90^\circ}{\sin 67^\circ}$ $AP = 19.55m$	✓Ratio ✓answer (2) ✓Ratio ✓answer (2)
7.2	$AB^2 = (22.65)^2 + (19.55)^2 - (2)(22.62)(19.55).\cos 42^\circ$ $AB^2 = 237.084...$ $AB = 15.40m$	✓Use of cosine rule ✓substitution ✓237.0847

		✓answer (4)
		[6]

Question 8

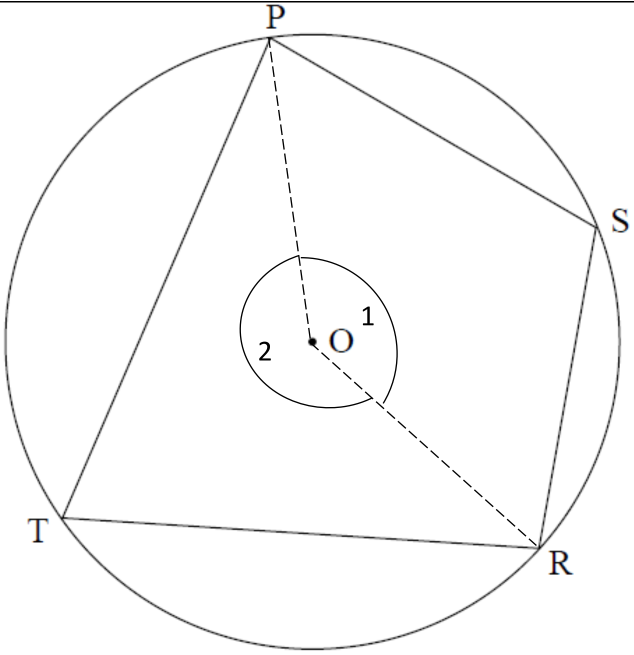
8.1		
8.1.1	$\hat{D}_1 = \hat{B}_1 = 40^\circ$ (tangent - chord theorem) $\therefore \hat{D}_2 = \hat{B}_1 = 40^\circ$ ($\angle$'s opp = sides)	✓S ✓R ✓S/R (3)
8.1.2	$\hat{C} = 180^\circ - (40^\circ + 40^\circ)$ ($\angle$s in Δ) $= 100^\circ$	✓S ✓S (2)
8.1.3	$\hat{A} = 180^\circ - 100^\circ$ (Opposite angles of a cyclic quad) $= 80^\circ$	✓R ✓S (2)
8.1.4	$\hat{O}_1 = 2\hat{A}$ ($\angle$ at centre $= 2 \times \angle$ at circun) $\hat{O}_1 = 160^\circ$ **OR**	✓R ✓S (2) ✓R

	$\hat{D}_2 = \hat{B}_1 = 40^\circ$ (From 9.1) $\hat{D}_3 = 90^\circ - 40^\circ - 40^\circ$ (tan ⊥ rad) $\hat{D}_3 = 10^\circ$ $\therefore \hat{O}_1 = 180^\circ - 10^\circ - 10^\circ$ (sum of angles in Δ) $\hat{O}_1 = 160^\circ$	✓S (2)
8.2		
8.2.1	Line from centre ⊥ to chord	✓R (1)
8.2.2 (a)	$\Delta CAP \parallel\parallel \Delta BDP$ $\frac{AP}{DP} = \frac{CP}{BP}$ (sides in proportion) $\frac{2t}{t} = \frac{15}{2t}$ $4t^2 = 15t$ $4t^2 - 15t = 0$ $2t(2t - 7.5) = 0$ $\therefore t \neq 0 \quad or \quad t = \frac{15}{4}$	✓S ✓S ✓S ✓answer (4) ✓S

	OR $15+t=2r$ $\therefore r=\frac{15+t}{2}$ $In\ \Delta \text{MPB}:$ $\left(\frac{15+t}{2}\right)^2=(2t)^2+\left(\frac{15+t}{2}-t\right)^2$ $\frac{225+30t+t^2}{4}=4t^2+\left(\frac{15-t}{2}\right)^2$ $\frac{225+30t+t^2}{4}=4t^2+\frac{225-30t+t^2}{4}$ $225+30t+t^2=16t^2+225-30t+t^2$ $16t^2-60t=0$ $4t(4t-15)=0$ $t=\frac{15}{4}$	✓S ✓S ✓answer (4)
	OR $15+t=2r$ $t=2r-15$ $In\ \Delta \text{MPB}:$ $\left(\frac{15+2r-15}{2}\right)^2=(2(2r-15))^2+\left(\frac{15+2r-15}{2}-(2r-15)\right)^2$ $r^2=4(4r^2-60r+225)+(r-2r+15)^2$ $r^2=16r^2-240r+900+(-r+15)^2$ $r^2=16r^2-240r+900+r^2-30r+225$ $0=16r^2-270r+1125$ $0=(2r-15)(8r-75)$ $\therefore r\neq\frac{15}{2}\quad or\quad r=\frac{75}{8}$ $\therefore r=\frac{15}{4}$	✓S ✓S ✓S ✓answer (4)
8.2.2 (b)	$\text{Radius}=\frac{15+\frac{15}{4}}{2}$ $=9.375\ \text{ OR }\ 9\frac{3}{8}$	✓method ✓answer (2)

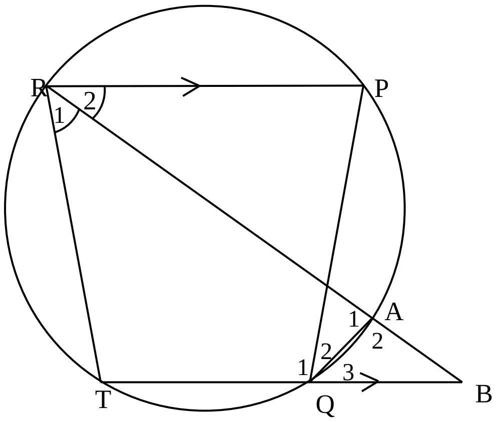
		[16]

Question 9

9.1	 Join PO and OR Let $\hat{O}_1 = 2x$ $\hat{T} = x$ ($\angle$ at circ centre = 2 $\angle$ at circumference) $\hat{O}_2 = 360^\circ - 2x$ ($\angle$s round a point) $\hat{S} = 180 - x$ ($\angle$at circ centre = 2 $\angle$ at circumference) $\hat{T} + \hat{S} = 180^\circ - x + x$ $\hat{T} + \hat{S} = 180^\circ$ $\therefore P\hat{T}R + P\hat{S}R = 180^\circ$	✓ construction ✓ S ✓R ✓ S ✓ S ✓ S (5)
9.2		

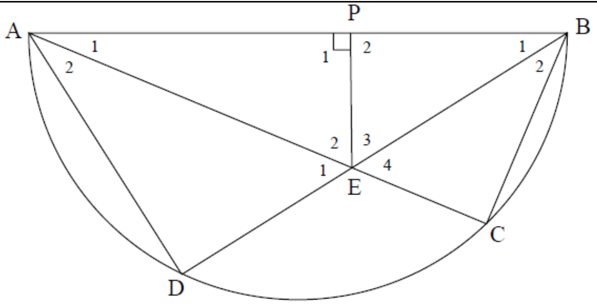
9.2.1	$\hat{R}_1 = x$ ($\angle$'s opp= radii) $\hat{O}_1 = 180^\circ - 2x$ ($\angle$sum in ΔQRT) $\hat{P}_1 = 90^\circ - x$ ($\angle$ circle centre = twice at circum)	✓S /R ✓S ✓S ✓R (4)
9.2.2	PQ = QR (given) $Q\hat{R}P = 90^\circ - x$ ($\angle$ opp=sides in Δ) $P\hat{Q}R = 2x$ ($\angle$ sum in ΔPQR) $x + \hat{Q}_2 = 2x$ $\hat{Q}_2 = x$ TQ bisects $P\hat{Q}R$	 ✓S ✓S ✓S (3)
9.2.3	$P\hat{Q}R = 2x$ $\hat{S} = 180^\circ - 2x$ (opp$\angle$'s of cyclic quad are suppl) $\hat{O}_1 = 180^\circ - 2x$ (FROM 10.2.1) STOR is a cyclic quad (converse – of ext $\angle$ of cyclic quad or ext $\angle$ of quad = int opp$\angle$) OR $P\hat{Q}R = 2x$ $\hat{S} = 180^\circ - 2x$ (opp$\angle$'s of cyclic quad are suppl) $\hat{O}_1 = 180^\circ - 2x$ (from 10.2.1) $\hat{O}_2 = 2x$ ($\angle$s on str. line) $\hat{S} + \hat{O}_2 = 180^\circ - 2x + 2x$ $\therefore \hat{S} + \hat{O}_2 = 180^\circ$ STOR is a cyclic quad (convse: opp $\angle$'s are supplementary)	✓S ✓S ✓R ✓S ✓R (5) ✓S ✓S ✓R ✓S ✓R (5)
		[17]

Question 10

		
10.1	$\hat{Q}_3 = \hat{R}_1 = x$ (ext angle of cyclic quad…) and (RA bisects $\hat{R}$) $\hat{R}_2 = \hat{Q}_2 = x$ (angles in the same segment) $\hat{R}_1 = \hat{R}_2$ $\therefore \hat{Q}_2 = \hat{Q}_3$ **OR** $\hat{Q}_2 + \hat{Q}_3 = \hat{R}_1 + \hat{R}_2$ (ext $\angle$ of cyclic quad) But $\hat{Q}_2 = \hat{R}_2$ ($\angle$s in same segment) $\hat{R}_1 = \hat{R}_2$ $\therefore \hat{Q}_3 = \hat{Q}_2$	✓S ✓R ✓S ✓R ✓S (5) ✓S ✓R ✓S ✓R ✓S (5)
10.2	$\hat{Q}_3 = \hat{B} = x$ ($\angle$s opp=sides, AQ = AB) $\hat{Q}_3 = \hat{R}_1 = \hat{B} = x$ (from 1.1) $\therefore$ TR = TB (sides opp = $\angle$s)	✓ S/R ✓R (2)
10.3	$\hat{P} = \hat{A}_1$ ($\angle$s in same segment) $\hat{A}_1 = \hat{Q}_3 + B$ (ext $\angle$ of ΔABC) $\hat{Q}_3 + \hat{B} = 2\hat{Q}_3$ ($\hat{Q}_3 = \hat{B}$ $\angle$s opp = sides) $2\hat{Q}_3 = 2\hat{R}_1$ (from 11.2) $\therefore 2\hat{R}_1 = P\hat{R}T$ $\therefore \hat{P} = T\hat{R}P$	✓S ✓R ✓S/R (3)

	OR	
	$T\hat{R}P = 2x$	✓ S ✓ R
	$\hat{A}_1 = \hat{Q}_3 + B = 2x$ (ext $\angle$ of ΔABC)	✓S/R
	$\hat{P} = \hat{A}_1 = 2x$ ($\angle$s in same segment)	(3)
	$= T\hat{R}P$	
		[10]

Question 11

		
11.1	*In* ΔBPE and ΔBDA	
	1. $\hat{B}_1 = \hat{B}_1$ common	✓S
	2. $\hat{P}_2 = \hat{D} = 90^\circ$ $\angle$ in semi – circle	✓S ✓R
	3. $B\hat{A}D = \hat{E}_3$ $\angle$ in Δ^s	
	$\therefore \Delta BPE \parallel\!\mid \Delta BDA$ (equiangular OR $\angle\angle\angle$)	✓R (4)
11.2	$\frac{BP}{BD} = \frac{BE}{AB}$ From Q11.1	✓ S/ratio
	$AB = \frac{BD.BE}{BP}$	✓S
	$AB^2 = \frac{BD^2.BE^2}{BP^2}$	✓S
	In Δ BPE;	
	$BE^2 = BP^2 + PE^2$ (pyth)	✓ $BE^2 = BP^2 + PE^2$
	$AB^2 = \frac{BD^2.(BP^2 + PE^2)}{BP^2}$	✓substitution
	$AB^2 = \frac{BD^2.BP^2}{BP^2} + \frac{BD^2.PE^2}{BP^2}$	✓simplification
	$AB^2 = BD^2 + \frac{BD^2.PE^2}{BP^2}$	(6)
		[10]
	TOTAL 150	