Education and Sport Development

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NORTH WEST PROVINCE

NATIONAL SENIOR CERTIFICATE/ *NASIONALE SENIOR SERTIFIKAAT*

GRADE/*GRAAD* 12

MATHEMATICS P2/*WISKUNDE V2*

SEPTEMBER 2018

MEMORANDUM

MARKS/*PUNTE*: 150

This memorandum consists of 22 pages./
Hierdie memorandum bestaan uit 22 bladsye.

NOTE:

- If a candidate answers a question TWICE, only mark the FIRST attempt.
- If a candidate has crossed out an attempt of a question and not redone the question, mark the crossed out version.
- Consistent accuracy applies in ALL aspects of the marking memorandum. Stop marking at the second calculation error.
- Assuming answers/values in order to solve a problem is NOT acceptable.

NOTA:

- *As 'n kandidaat 'n vraag TWEE KEER beantwoord, merk slegs die EERSTE poging.*
- *As 'n kandidaat 'n antwoord van 'n vraag doodtrek en nie oordoen nie, merk die doodgetrekte poging.*
- *Volgehoue akkuraatheid word in ALLE aspekte van die nasienmemorandum toegepas. Hou op nasien by die tweede berekeningsfout.*
- *Aanvaar van antwoorde of waardes om 'n probleem op te los, word NIE toegelaat nie.*

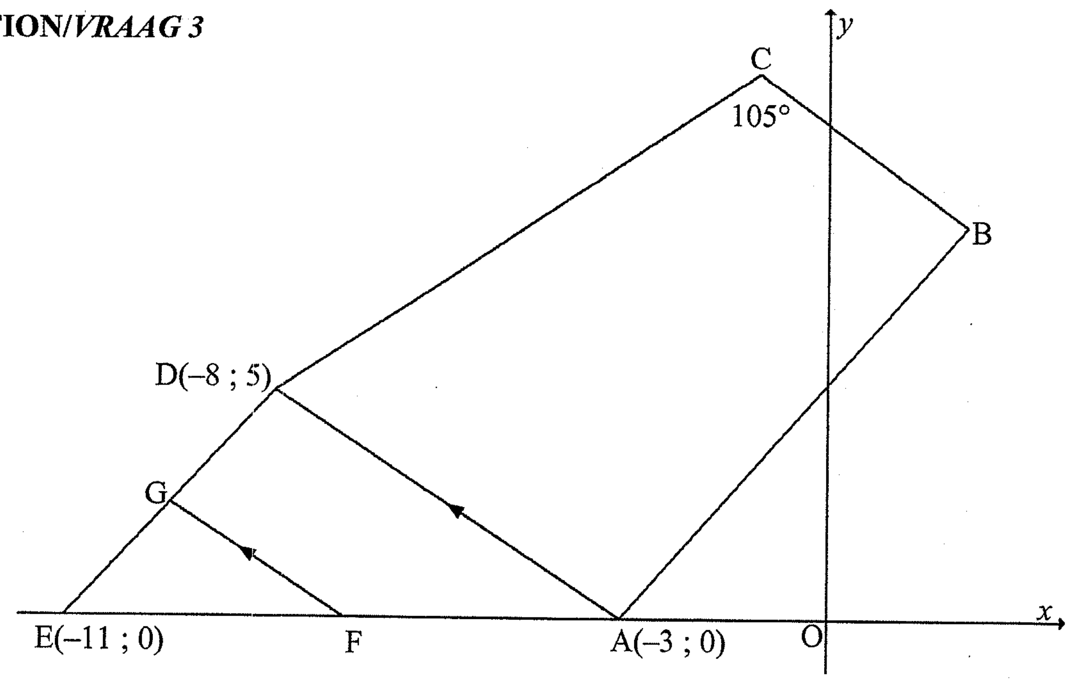
QUESTION/*VRAAG 1*

1.1	$\bar{x} = \frac{5478}{11}$ $\bar{x} = 498$ NOTE: ANSWER ONLY, FULL MARKS	✓ $\frac{5478}{11}$ ✓ 498 (2)
1.2	$\sigma_x = 119{,}47$	✓✓answer/*antwoord* (2)
1.3	$(\bar{x} - \sigma_x \; ; \bar{x} + \sigma_x)$ $(498 - 119{,}47 \; ; 498 + 119{,}47)$ $(378{,}53 \; ; 617{,}47)$ $\therefore$ 5 distances / *afstande*	✓ 378,53 ✓ 617,47 ✓ 5 (3)
1.4.1	new average / *nuwe gemiddeld* : $\bar{k} = \frac{5555}{11}$ $\bar{k} = 505$ the value of y / *die waarde van y* : $y = \bar{k} - \bar{x} = 505 - 498 = 7\ cm$ **OR / *OF*** $11y = 5555 - 5478$ $y = \frac{77}{11}$ $y = 7\ cm$	✓ $\bar{k} = 505$ ✓ $y = 7$ **OR/*OF*** ✓ $11y = 5555 - 5478$ ✓ $y = 7$ (2)
1.4.2	119,47	✓✓correct answer / *korrekte antwoord* (2) **[11]**

QUESTION/*VRAAG* 2

2.1	$\hat{y} = a + bx$ $a = 57,87$ and / *en* $b = 0,05$ $\therefore \hat{y} = 57,87 + 0,05x$	✓ $a = 57,87$ ✓ $b = 0,05$ ✓ equation/ *vergelyking* (3)
2.2	$r = 0,93$	✓✓ 0,93 (2)
2.3	Strong positive correlation / *Sterk positiewe korrelasie*	✓Strong / *sterk* ✓Positive / *positief* (2)
2.4	$\hat{y} = 57,87 + 0,05(465)$ $\hat{y} = 81,12\%$ **OR / *OF*** $\hat{y} = 80,81\%$ (calculator / *sakrekenaar*) $\approx 81\%$	✓ substitute 465 into eq. / *vervang 465 in vgl.* ✓ 81,12 (2) ✓✓ 80,81% / 81% **[9]**

QUESTION/*VRAAG* 3



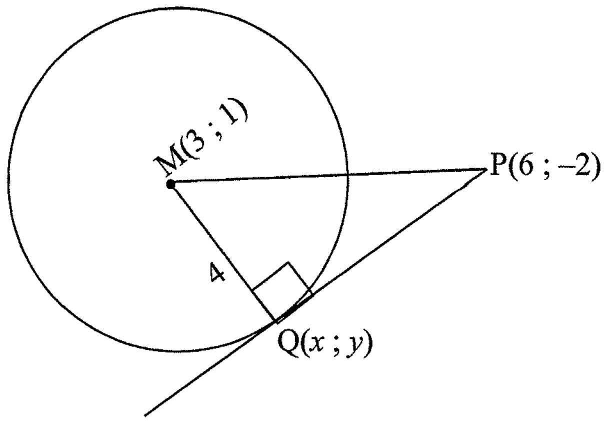
3.1	Length of AD / *Lengte van AD*: $d_{AD} = \sqrt{(-3+8)^2+(0-5)^2}$ $d_{AD} = \sqrt{50}$ $d_{AD} = 5\sqrt{2}$ or / *of* 7,07 units / *eenhede* Length of DE / *Lengte van DE*: $d_{DE} = \sqrt{(-8+11)^2+(5-0)^2}$ $d_{DE} = \sqrt{34}$ or / *of* 5,83 units / *eenhede* Length of AE / *Lengte van AE* = 8 units / *eenhede* Perimeter / *Omtrek* Δ ADE $= 5\sqrt{2}+\sqrt{34}+8 = 20,9$ *units* / *eenhede* or / *of* $7,07+5,83+8 = 20,90$	✓ $d_{AD} = \sqrt{(-3+8)^2+(0-5)^2}$ ✓ $d_{AD} = 5\sqrt{2}$ or / *of* 7,07 ✓ $d_{DE} = \sqrt{34}$ or / *of* 5,83 ✓ AE = 8 ✓ $5\sqrt{2}+\sqrt{34}+8$ or/*of* 7,07 + 5,83 + 8 (5)
3.2.1	Line from mdpt 1st side of Δ, parallel to 2nd side, bisects 3rd side / *Lyn uit midpt van een sy van* Δ, *ewewydig aan 2de sy, halveer 3de sy.*	✓reason/*rede* (1)
3.2.2	$x_G = \frac{-11+(-8)}{2}$ and/*en* $y_G = \frac{0+5}{2}$ $\therefore x_G = -\frac{19}{2}$ $\therefore y_G = \frac{5}{2}$ $\therefore G\left(-\frac{19}{2};\frac{5}{2}\right)$	✓ method/*metode* ✓ $x_G = -\frac{19}{2}$ ✓ $y_G = \frac{5}{2}$ (3)
3.2.3	$FG = \frac{1}{2}AD = \frac{5\sqrt{2}}{2}$ or / *of* 3,54 units / *eenhede*	✓ answer/*antwoord* (1)

3.2.4	$m_{AD} = \frac{0-5}{-3-(-8)}$ $= \frac{-5}{5}$ $= -1$ $m_{FG} = m_{AD}$ $[AD \parallel FG]$ Through the point / *Deur die punt* $\left(-\frac{19}{2};\frac{5}{2}\right)$ $y-\frac{5}{2} = -1\left(x+\frac{19}{2}\right)$ $y = -x-\frac{19}{2}+\frac{5}{2}$ $\therefore y = -x-7$	✓ $m_{FG} = m_{AD}$ $[AD \parallel FG]$ ✓ $m_{FG} = -1$ ✓Subst./*Vervang* $\left(-\frac{19}{2};\frac{5}{2}\right)$ ✓ $y = -x-7$ (4)
3.3	$\tan D\hat{A}O = m_{AD}$ $\tan D\hat{A}O = -1$ $\therefore D\hat{A}O = -45°+180°$ $D\hat{A}O = 135°$	✓ $\tan D\hat{A}O = -1$ ✓ $D\hat{A}O = 135°$ (2)

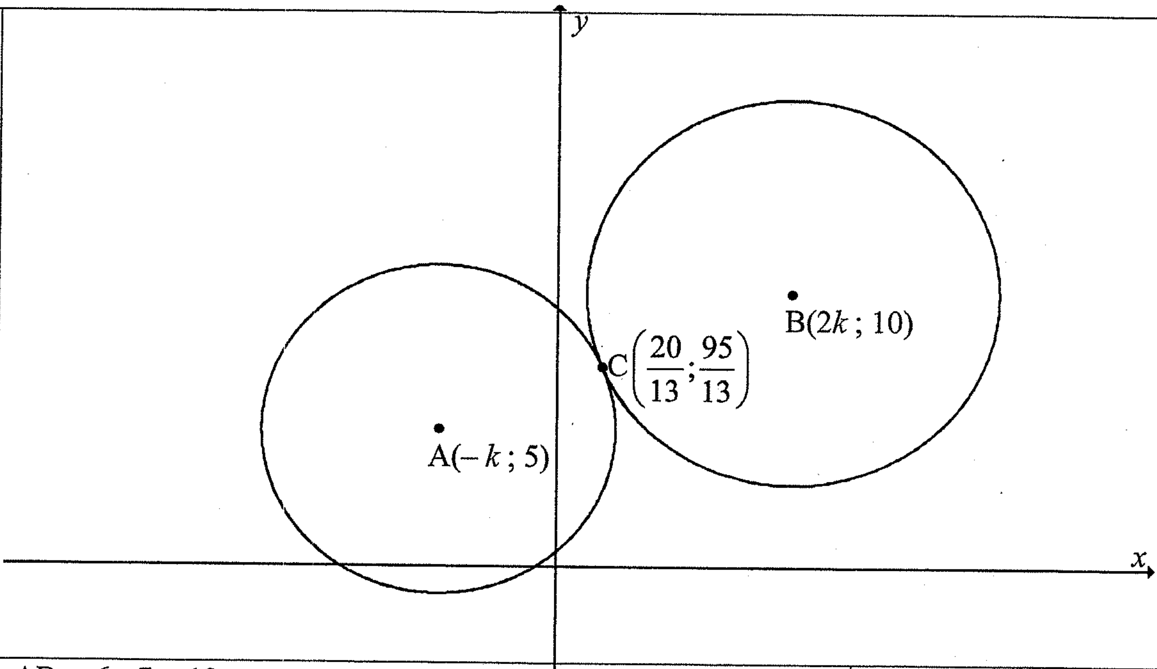
3.4	$D\hat{A}B = 75°$ $[opp.\angle s\ cyclic\ quad. /\ teenoorst.\angle e\ kvh]$ $O\hat{A}B = 135° - 75°$ $= 60°$ $\tan O\hat{A}B = m_{AB}$ $\tan 60° = m_{AB} = \sqrt{3}$ equation / *vergelyking AB* through / *deur* $(-3;0)$: $y - 0 = \sqrt{3}(x+3)$ $y = \sqrt{3}x + 3\sqrt{3}$ at / *by B*: $\frac{-12+5\sqrt{3}}{3}x + \frac{24+5\sqrt{3}}{3} = \sqrt{3}x + 3\sqrt{3}$ $\frac{-12+5\sqrt{3}}{3}x - \sqrt{3}x = 3\sqrt{3} - \frac{24+5\sqrt{3}}{3}$ $x\left(\frac{-12+5\sqrt{3}}{3} - \sqrt{3}\right) = 3\sqrt{3} - \frac{24+5\sqrt{3}}{3}$ $x = 2$ and/*en* $\therefore y = \sqrt{3}(2) + 3\sqrt{3}$ **OR / *OF*** $y = \frac{-12+5\sqrt{3}}{3}(2) + \frac{24+5\sqrt{3}}{3}$ $\therefore y = 5\sqrt{3}$ $\therefore B(2;5\sqrt{3})$	✓ $D\hat{A}B = 75°$ ✓ $O\hat{A}B = 60°$ ✓ $m_{AB} = \sqrt{3}$ ✓ $y = \sqrt{3}x + 3\sqrt{3}$ ✓ $\frac{-12+5\sqrt{3}}{3}x + \frac{24+5\sqrt{3}}{3} = \sqrt{3}x + 3\sqrt{3}$ ✓ $x = 2$ ✓ $y = 5\sqrt{3}$ (7) **[23]**

QUESTION/*VRAAG* 4

4.1	$x^2 + y^2 = r^2$ $x^2 + y^2 = (2\sqrt{3})^2$ $x^2 + y^2 = 12$	✓ subst. into eq./*vervang in vgl.* ✓ answer/*antwoord* (2)
4.2.1	$x^2 - 6x + (-3)^2 + y^2 - 2y + (-1)^2 = 6 + (-3)^2 + (-1)^2$ $\therefore (x-3)^2 + (y-1)^2 = 16$	✓ $(x-3)^2 + (y-1)^2$ ✓ 16 (2)
4.2.2	M(3 ; 1)	✓ $x = 3$ ✓ $y = 1$ (2)
4.2.3	4	✓answer/*antwoord* (1)

4.2.4		✓MQ ⊥ PQ
	$d_{MP} = \sqrt{(6-3)^2+(-2-1)^2}$ $d_{MP} = \sqrt{18}$ $d_{MP} = 3\sqrt{2}$ Pythagoras : $PQ^2 = (3\sqrt{2})^2 - 4^2$ $PQ^2 = 18 - 16$ $\therefore PQ = \sqrt{2}$ units / *eenhede*	✓ $d_{MP} = \sqrt{18}$ or / *of* $3\sqrt{2}$ ✓$PQ^2 = (3\sqrt{2})^2 - 4^2$ ✓$PQ = \sqrt{2}$ (4)

4.3



4.3.1

AB = 6 + 7 = 13

$d_{AB} = \sqrt{(2k-(-k))^2+(10-5)^2}$

$13^2 = (3k)^2 + (5)^2$

$169 = 9k^2 + 25$

$144 = 9k^2$

$16 = k^2$

$k = \pm 4$

$\therefore k = 4$ (see diagram / *sien diagram*)

$\therefore A(-4;5)$ *and / en* $B(8;10)$

Gradient of *AB* /
Gradiënt van AB

$m_{AB} = \dfrac{\frac{95}{13}-5}{\frac{20}{13}-(-4)}$ or / *of* $m_{AB} = \dfrac{\frac{95}{13}-10}{\frac{20}{13}-8}$ *or / of* $m_{AB} = \dfrac{10-5}{8+4} = \dfrac{5}{12}$

$m_{AB} = \dfrac{5}{12}$

OR/*OF*

Marks:

✓ AB = 6 + 7 = 13

✓ $13^2 = (3k)^2 + (5)^2$

✓ $k^2 = 16$

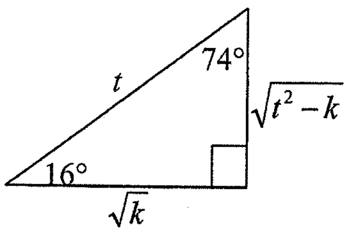
✓ $k = 4$

✓ Correct subst. into gradient / *vervang korrek in gradiënt*

(5)

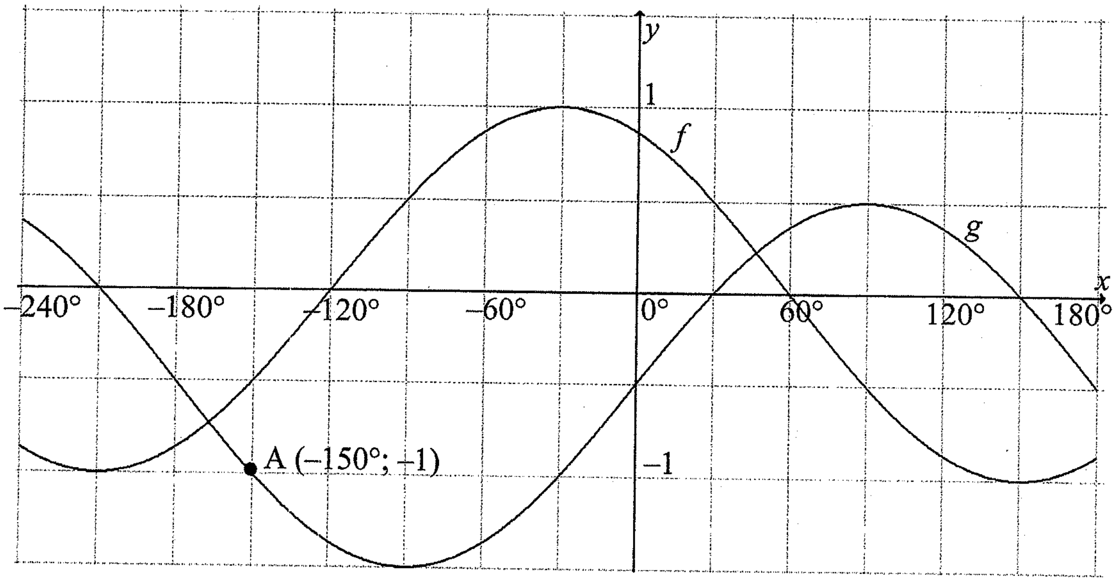
	$M_{AB}=\frac{5}{3k}$ $M_{AC}=\frac{\frac{95}{13}-5}{\frac{20}{13}+k}$ $=\frac{95-65}{20+13k}$ $\frac{5}{3k}=\frac{30}{20+13k}$ $100+65k=90k$ $100=25k$ $k=4$ $\therefore M_{AB}=\frac{5}{3(4)}=\frac{5}{12}$	✓ $M_{AB}=\frac{5}{3k}$ ✓ $M_{AB}=\frac{\frac{95}{13}-5}{\frac{20}{13}+k}$ ✓ $\frac{5}{3k}=\frac{95-65}{20+13k}$ ✓ $k=4$ ✓ Correct subst. into gradient / *vervang korrek in gradiënt* (5)
4.3.2	radius ⊥ tangent / *radius ⊥ raaklyn* $\therefore m_{\text{tangent/raaklyn}}=-\frac{12}{5}$ Equation of common tangent / *Vergelyking van gemeenskaplike raaklyn* $y-\frac{95}{13}=-\frac{12}{5}\left(x-\frac{20}{13}\right)$ $\therefore y=-\frac{12}{5}x+11$	✓ $m_{\text{tangent/raaklyn}}=-\frac{12}{5}$ ✓ Subst. / *vervang* $\left(\frac{20}{13};\frac{95}{13}\right)$ ✓ $y=-\frac{12}{5}x+11$ (3) **[19]**

QUESTION/*VRAAG 5*

5.1.1	$\sin 106°$ $= \sin(180° - 74°)$ $= \sin 74°$ $= \cos 16°$ $= \frac{\sqrt{k}}{t}$	✓ $\cos 16°$ ✓ $\frac{\sqrt{k}}{t}$ (2)
5.1.2	$\sin 16° = \frac{\sqrt{t^2 - k}}{t}$  **OR/*OF*** $\sin^2 16° + \cos^2 16° = 1$ $\sin^2 16° + \left(\frac{\sqrt{k}}{t}\right)^2 = 1$ $\sin^2 16° = 1 - \frac{k}{t^2}$ $\sin^2 16° = \frac{t^2 - k}{t^2}$ $\sin 16° = \frac{\sqrt{t^2 - k}}{t}$	make use of diagram/ *maak van diagram gebruik* ✓✓ $\sqrt{t^2 - k}$ ✓ $\frac{\sqrt{t^2 - k}}{t}$ **OR/*OF*** ✓ $\sin^2 16° + \cos^2 16° = 1$ ✓ $1 - \frac{k}{t^2}$ ✓ $\frac{\sqrt{t^2 - k}}{t}$ (3)
5.1.3	$\cos 16° = \cos 2(8°) = 2\cos^2(8°) - 1$ $\frac{\sqrt{k}}{t} = 2\cos^2(8°) - 1$ $\frac{\sqrt{k}}{t} + 1 = 2\cos^2(8°)$ $\frac{\sqrt{k}}{2t} + \frac{1}{2} = \cos^2(8°)$ $\therefore \cos 8° = \sqrt{\frac{\sqrt{k}}{2t} + \frac{1}{2}} = \sqrt{\frac{\sqrt{k} + t}{2t}}$	✓ $\cos 2(8°)$ ✓ $\cos 16° = 2\cos^2(8°) - 1$ ✓ $\frac{\sqrt{k}}{t} = 2\cos^2(8°) - 1$ ✓ $\sqrt{\frac{\sqrt{k}}{2t} + \frac{1}{2}}$ or / *of* $\sqrt{\frac{\sqrt{k} + t}{2t}}$ (4)

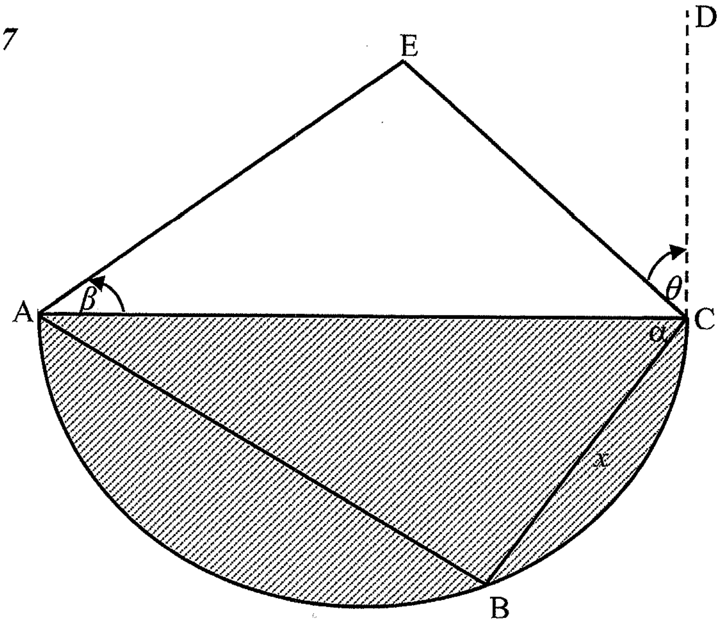
5.2	LHS / *LK* $\dfrac{\sqrt{4(1-\cos\theta)(1+\cos\theta)}}{\sin 2\theta}$ $=\dfrac{\sqrt{4(1-\cos^2\theta)}}{2\sin\theta\cos\theta}$ $=\dfrac{2\sqrt{\sin^2\theta}}{2\sin\theta\cos\theta}$ $=\dfrac{2\sin\theta}{2\sin\theta\cos\theta}$ $=\dfrac{1}{\cos\theta}$ = RHS / *RK*	✓ $(1-\cos\theta)(1+\cos\theta)$ $=1-\cos^2\theta$ ✓ $1-\cos^2\theta=\sin^2\theta$ ✓ $\sin 2\theta = 2\sin\theta\cos\theta$ ✓ $\dfrac{2\sin\theta}{2\sin\theta\cos\theta}$ (4)
5.3.1	$\sin p+\sqrt{3}\cos p = 1$ $\frac{1}{2}\sin p+\frac{\sqrt{3}}{2}\cos p = \frac{1}{2}$ $(\div 2)$ $\cos 60°\sin p+\sin 60°\cos p = \frac{1}{2}$ (special angles / *spesiale hoeke*) $\sin(60°+p) = \frac{1}{2}$ (compound angles / *saamgestelde hoeke*)	✓ $\frac{1}{2}\sin p+\frac{\sqrt{3}}{2}\cos p = \frac{1}{2}$ ✓ $\cos 60°\sin p$ ✓ $\sin 60°\cos p$ (3)
5.3.2	$\sin(60°+p) = \frac{1}{2}$ ref.∠ / *verw.*∠ = 30° $60°+p = 30°+n.360°$ or / *of* $60°+p = (180°-30°)+n.360°$ $p = -30°+n.360°$ or / *of* $p = 90°+n.360°$ $(n\in Z)$	✓ ref.∠ / *verw.*∠ = 30° ✓ $p = -30°+n.360°$ / $p = 330°+n.360°$ ✓ $p = 90°+n.360°$ ✓ $(n\in Z)$ if used in correct context / *as dit in korrekte konteks gebruik is* (4)
5.4	$\cos 0°+\cos 1°+\cos 2°+...+\cos(180-2)°+\cos(180-1)°+\cos 180°+2$ $1+\cos 1°+\cos 2°...+(-\cos 2°)+(-\cos 1°)+(-1)+2$ $1+(-1)+2$ $=2$	✓ $\cos(180-2)°+\cos(180-1)°$ ✓ $...(-\cos 2°)+(-\cos 1°)$ ✓ 2 (3) **[23]**

QUESTION/*VRAAG* 6



6.1	$p = 30°$ and/*en* $q = -\frac{1}{2}$	✓✓ $p = 30°$ ✓✓ $q = -\frac{1}{2}$ (4)
6.2	$x = -150°$ or/*of* $x = 30°$	✓ $-150°$ ✓ $30°$ (2)
6.3	The graph of *f* must be translated 60° to the left / *Die grafiek van f moet 60° na links getransleer word.* **OR/*OF*** The graph of *f* must be translated 120° to the right and then be reflected about the *x*-axis/ *Die grafiek van f moet 120° na regs getransleer word en dan in die x-as gereflekteer word.*	✓✓60° to the left/*na links* (2) ✓ 120° right/*regs* ✓ reflect about *x*-axis/reflekteer in die *x*-as. **[8]**

QUESTION/*VRAAG* 7

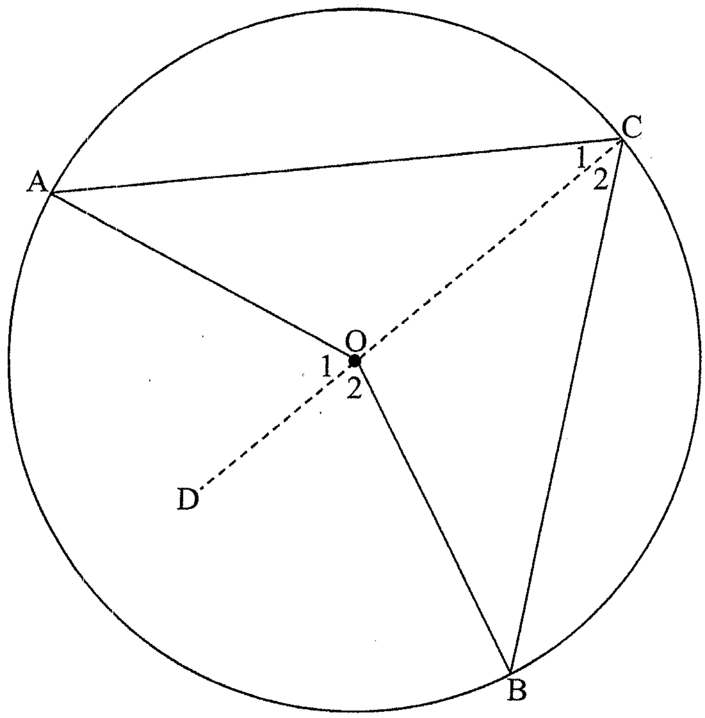


7.1	$\hat{B} = 90°$ $\cos\alpha = \frac{BC}{AC} = \frac{x}{AC}$ $\therefore AC = \frac{x}{\cos\alpha}$	✓ $\hat{B} = 90°$ ✓ $\cos\alpha = \frac{BC}{AC} = \frac{x}{AC}$ (2)
7.2	$E\hat{C}A = 90° - \theta$ $A\hat{E}C = 180° - \beta - (90° - \theta)$ $A\hat{E}C = 90° - \beta + \theta$ or / *of* $90° - (\beta - \theta)$ or / *of* $90° + (\theta - \beta)$	✓ $E\hat{C}A = 90° - \theta$ ✓ $A\hat{E}C = 180° - \beta - (90° - \theta)$ ✓ $A\hat{E}C = 90° - \beta + \theta$ or / *of* $90° - (\beta - \theta)$ or / *of* $90° + (\theta - \beta)$ (3)
7.3	In Δ ABC $AC = \frac{x}{\cos\theta}$ (see / *sien* 7.1) In Δ ACE $\frac{\sin\beta}{EC} = \frac{\sin A\hat{E}C}{\frac{x}{\cos\alpha}}$ $EC = \frac{\frac{x}{\cos\alpha} \times \sin\beta}{\sin[90° - (\beta - \theta)]}$ $EC = \frac{\frac{x\sin\beta}{\cos\alpha}}{\cos(\beta - \theta)}$ $= \frac{x\sin\beta}{\cos\alpha.\cos(\beta - \theta)}$	✓ Correct application of sine-rule/ *korrekte toepassing van sinus-reël* ✓ $EC = \frac{\frac{x}{\cos\alpha} \times \sin\beta}{\sin[90° - (\beta - \theta)]}$ ✓ $\sin[90° - (\beta - \theta)] = \cos(\beta - \theta)$ (3) **[8]**

GEOMETRY/*MEETKUNDE*	
S	**A mark for a correct statement (A statement mark is independent of a reason.)**
	'n Punt vir 'n korrekte bewering ('n Punt vir 'n bewering is onafhanklik van die rede.)
R	**A mark for a correct reason (A reason mark may only be awarded if the statement is correct.)**
	'n Punt vir 'n korrekte rede ('n Punt word slegs vir die rede toegeken as die bewering korrek is.)
S/R	**Award a mark if the statement AND reason are both correct.**
	Ken 'n punt toe as beide die bewering EN rede korrek is.

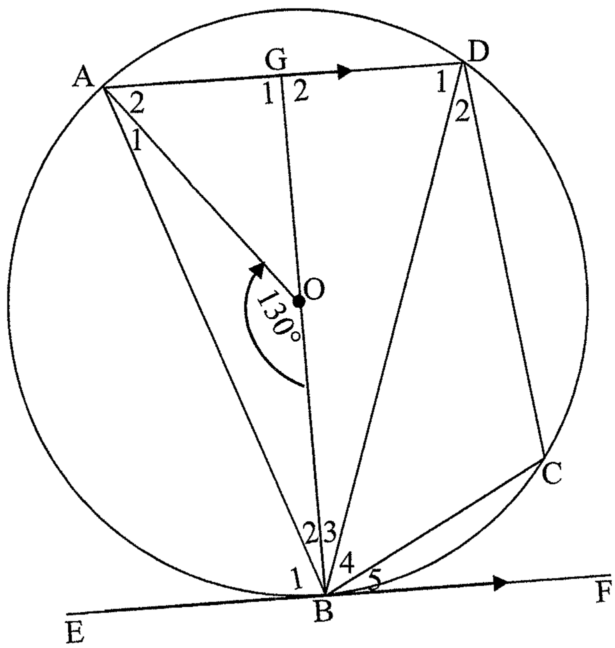
QUESTION/*VRAAG* 8

8.1



8.1	$\hat{O}_1 = \hat{C}_1 + \hat{A}$ [ext.∠ of Δ / *buite∠ van Δ*]	✓S/R
	But / *maar* $\hat{C}_1 = A$ [∠s opp. equal radii / *∠e teenoor gelyke radiusse*]	✓S/R
	$\therefore \hat{O}_1 = 2\hat{C}_1$	✓S
	In the same manner / *op dieselde wyse*:	
	$\hat{O}_2 = 2\hat{C}_2$	✓S
	$\hat{O}_1 + \hat{O}_2 = 2\hat{C}_1 + 2\hat{C}_2$	
	$\therefore A\hat{O}B = 2\hat{C}_1 + 2\hat{C}_2$	
	$= 2(\hat{C}_1 + \hat{C}_2)$	✓S
	$= 2A\hat{C}B$	(5)

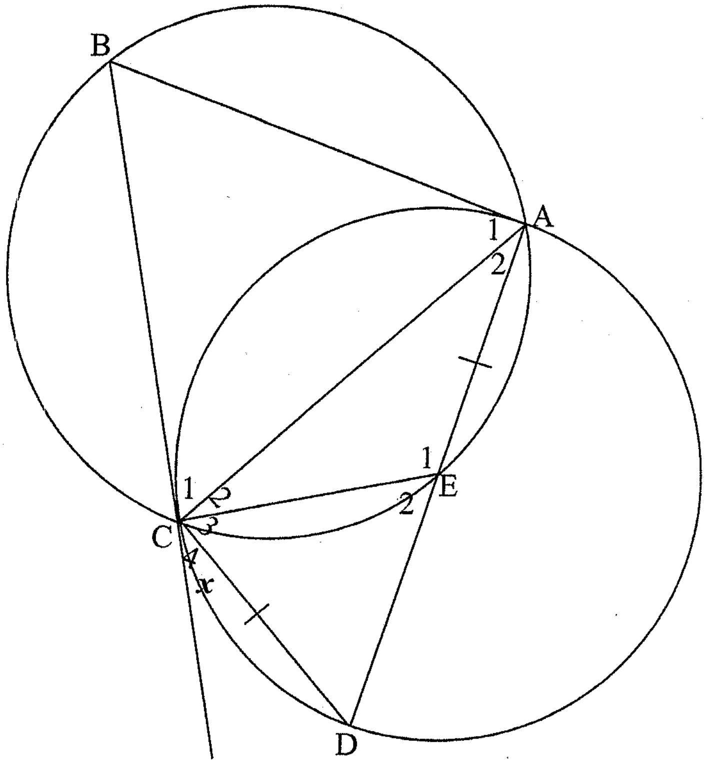
8.2



8.2.1	$\hat{D}_1 = 65°$ [∠ at centre = 2 × ∠ at circumference / *midpt.∠ = 2 × omtreks∠*]	✓S✓R (2)
8.2.2	$\hat{B}_1 = 65°$ [tangent chord theorem / *raaklyn – koordstelling*]	✓S✓R (2)
8.2.3	$B\hat{A}D = 65°$ [alt ∠s / *verwiss ∠e; AD // EF*]	✓S/R (1)
8.2.4	$\hat{C} = 115°$ [opp∠s of cyclic quad / *teenoorst.∠e van kvh*]	✓S✓R (2)
8.2.5	$G\hat{B}F = 90°$ [rad ⊥ tangent / *rad ⊥ raaklyn*] $D\hat{B}F = 65°$ [alt∠s / *verwiss ∠e; AD // EF*] $\hat{B}_3 = 90° - 65° = 25°$ **OR/*OF*** $\hat{B}_1 + \hat{B}_2 = 90°$ [rad ⊥ tangent / *rad ⊥ raaklyn*] $\hat{G}_2 = 90°$ [alt∠s / *verwiss ∠e; AD // EF*] $\hat{B}_3 = 180° - 65° - 90° = 25°$[∠*s of* Δ / *binne ∠e van* Δ)	✓S/R ✓S/R ✓S ✓S/R ✓S/R ✓S (3)

8.2.6	$\hat{G}_2 = 90°$ [co-interior ∠s / *ko – binne∠e; AD // EF*] / [Proved in 8.2.5]
	AG = GD [line from centre ⊥ chord; *Loodlyn vanuit midpt sirkel na koord*]
	$\therefore GD = \frac{\sqrt{7}}{4}$
		✓S/R
		✓S/R
		✓ $\frac{\sqrt{7}}{4}$
		(3)
		[18]

QUESTION/*VRAAG* 9

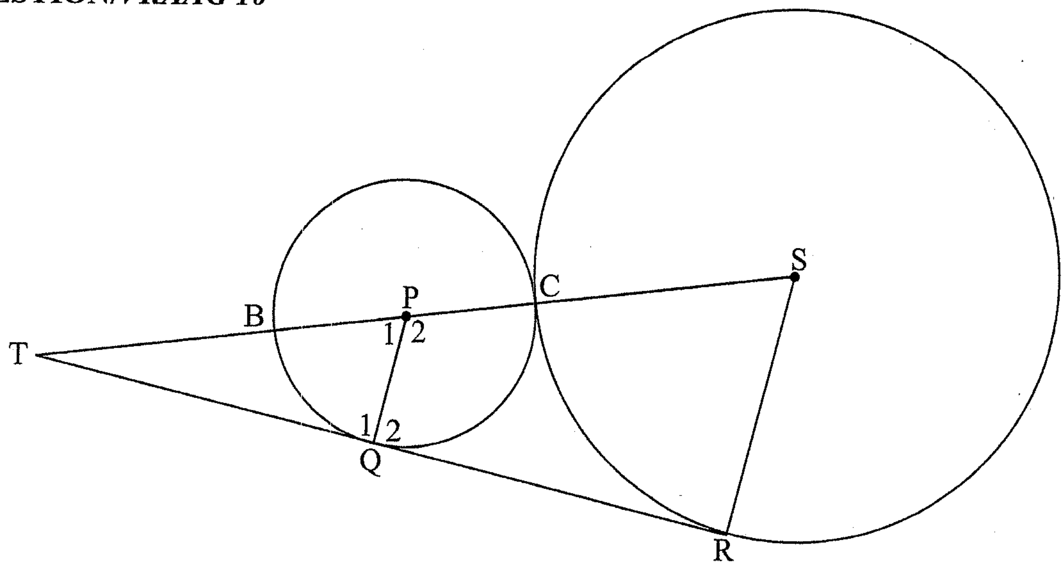


Let/*Laat* $\hat{C}_4 = x$.

9.1	$\hat{C}_4 = \hat{A}_2 = x$ [tangent chord theorem / *raaklyn – koordstelling*] $\hat{C}_2 = \hat{A}_2 = x$ [equal circles, equal chords, equal ∠s / *gelyke sirkels, gelyke koorde, gelyke∠e*] $\therefore \hat{C}_2 = \hat{C}_4$	✓S✓R ✓S✓R (4)
9.2	$\hat{C}_4 + \hat{C}_3 = \hat{A}_1 + \hat{A}_2$ [ext.∠s of cyclic quad / *buite∠e van kvh*] But / *Maar* $\hat{C}_4 = \hat{A}_2 = x$ [in 9.1] $\therefore \hat{C}_3 = \hat{A}_1$ **OR / OF** $\hat{E}_1 = 180° - 2x$ [sum of ∠s of Δ / *som van binne∠e van Δ*] $\hat{B} = 180° - (180° - 2x) = 2x$ [opp∠s of cyclic quad / *teenoorst.∠e van kvh*] $\hat{C}_2 + \hat{C}_3 + \hat{C}_4 = \hat{B} + \hat{A}_1$ [ext.∠ of Δ / *buite∠ van Δ*] But / *Maar* $\hat{C}_2 + \hat{C}_4 = x + x = 2x = \hat{B}$ $\therefore \hat{C}_3 = \hat{A}_1$	✓S✓R ✓S ✓S/R ✓S/R ✓S (3)

9.3	$\hat{C}_2 = \hat{A}_2$ [see/*sien* 9.1] $\therefore$ CE = AE [sides opp. = ∠s / *sye teenoor* = ∠*e*] $\hat{A}_1 = \hat{D}$ [tan-chord theorem / *raaklyn – koord – stelling*] $\hat{A}_1 = \hat{C}_3$ [from / *van* 9.2] $\therefore \hat{D} = \hat{C}_3$ $\therefore$ CE = DE [sides opp. = ∠s / *sye teenoor* = ∠*e*] CE = AE = DE $\therefore$ E is the centre of the circle / *is die middelpunt van die sirkel*	✓S/R ✓S/R ✓S/R ✓S (4)
9.4	AE = EC = ED [radii / *radiusse*] But / *Maar* CD = AE [Given / *gegee*] $\therefore$ CD = DE = CE $\therefore \Delta$ECD is equilateral / *gelyksydig*	✓S ✓S (2) **[13]**

QUESTION/*VRAAG 10*



10.1	Parallel to the third side of the Δ / *Ewewydig aan die derde sy van die* Δ	✓R (1)
10.2.1	$\hat{R} = 90°$ [tangent $\perp$ radius / *raaklyn* $\perp$ *radius*] $\hat{Q}_1 = 90°$ [tangent $\perp$ radius / *raaklyn* $\perp$ *radius*] $\hat{Q}_1 = \hat{R} = 90°$ $\therefore$ PQ // SR [corresponding $\angle$s are equal / *ooreenk.* $\angle$*e is gelyk*] **OR / *OF*** $\hat{R} = 90°$ [tangent $\perp$ radius / *raaklyn* $\perp$ *radius*] $\hat{Q}_2 = 90°$ [tangent $\perp$ radius / *raaklyn* $\perp$ *radius*] $\therefore \hat{R} + \hat{Q}_2 = 180°$ $\therefore$ PQ $\parallel$ SR [co-interior $\angle$s = 180°; *ko-binne* $\angle$*e saam* 180°]	✓S ✓R ✓S/R ✓R ✓S ✓R ✓S/R ✓R (4)
10.2.2	$\frac{TP}{PS} = \frac{TQ}{QR}$ [prop.theorem; PQ $\parallel$ SR / *eweredigheidst.; PQ* $\parallel$ *SR*] $\therefore TP = \frac{TQ.PS}{QR}$ PC = BP and / *en* SC = SR [radii / *radiusse*] PS = PC + CS = BP + SR $\therefore TP = \frac{TQ(BP+SR)}{QR}$	✓S ✓R ✓S/R ✓S (4)

10.2.3	In ΔTQP and / *en* ΔTRS $\hat{T} = \hat{T}$ [common / *gemeenskaplik*] $\hat{Q}_1 = \hat{R}$ [corresp.$\angle$s / *ooreenk.$\angle$e*; PQ || SR] $\hat{P}_1 = \hat{S}$ [sum of $\angle$s Δ/*som van $\angle$e Δ*] $\therefore \Delta TQP \parallel\!\mid \Delta TRS$ **OR / *OF*** $\hat{T} = \hat{T}$ [common / *gemeenskaplik*] $\hat{Q}_1 = \hat{R}$ *or / of* $\hat{P}_1 = \hat{S}_1$ [corresp.$\angle$s / *ooreenk.$\angle$e*; PS || QR] $\therefore \Delta TQP \parallel\!\mid \Delta TRS$ [$\angle,\angle,\angle$]	✓S ✓S ✓S ✓S ✓S ✓R (3)
10.2.4	$TS^2 - SR^2 = TR^2$ [Pythagoras] $CS = SR$ [radii / *radiusse*] $\therefore TS^2 - CS^2 = TR^2$ $\frac{TQ}{TR} = \frac{QP}{RS} = \frac{TP}{TS}$ [$\parallel\!\mid \Delta$s] $TR = \frac{TQ.RS}{QP}$ But / *Maar* QP = BP and/en RS=CS [radii/*radiusse*] $\therefore TR = \frac{TQ.CS}{BP}$ $TQ^2 = TP^2 + PQ^2 - 2TP.PQ.\cos P_1$ But/*Maar* PQ = BP and/*en* $\hat{S}_1 = \hat{P}_1$ $\therefore TQ^2 = (TP^2 + BP^2 - 2TP.BP\cos S)$ $TR^2 = \frac{TQ^2CS^2}{BP^2}$ $\therefore TS^2 - CS^2 = \frac{TQ^2CS^2}{BP^2}$ $\therefore \sqrt{TS^2 - CS^2} = \frac{\sqrt{(TP^2 + BP^2 - 2TP.BP\cos S)}.CS}{BP}$	✓S ✓S/R ✓S ✓use cosine-rule to find TQ/ *Gebruik cosinus-reël om* TQ *te vind* ✓ $\frac{TQ^2CS^2}{BP^2}$ ✓ $TS^2 - CS^2 = \frac{TQ^2CS^2}{BP^2}$ (6) **[18]**

TOTAL/*TOTAAL*: 150