LIMPOPO
PROVINCIAL GOVERNMENT
REPUBLIC OF SOUTH AFRICA

DEPARTMENT OF
EDUCATION

NATIONAL SENIOR CERTIFICATE

GRADE 12

MATHEMATICS PAPER 2

SEPTEMBER 2017

MARKS: 150

TIME: 3 HOURS

This question paper consists of 11 pages, an information sheet and a 15-pages answer book.

INSTRUCTIONS AND INFORMATION

Read the following instructions carefully before answering the questions.

1. This question paper consists of 10 questions.
2. Answer ALL the questions in the ANSWER BOOK provided.
3. Answer ALL the questions.
4. Clearly show ALL calculations, diagrams, graphs, et cetera that you have used in determining your answers.
5. ANSWERS ONLY will not necessarily be awarded full marks.
6. You may use an approved scientific calculator (non-programmable and non-graphical), unless stated otherwise.
7. If necessary, round answers off to TWO decimal places, unless stated otherwise.
8. Diagrams are NOT necessarily drawn to scale.
9. Number the answers correctly according to the numbering system used in this question paper.
10. Write legibly and present your work neatly.

QUESTION 1

A class of 25 learners was given a test out of 25. The marks are as follows:

MARKS OBTAINED	FREQUENCY
8	2
10	3
12	4
17	5
20	5
22	4
23	2

1.1 Calculate the mean of the data. (2)

1.2 Determine the 1.2.1 median (2)

1.2.2 standard deviation of the data. (2)

1.3 Determine how many of the scores will be outside ONE standard deviation from the mean. Show all the necessary calculations. (3)

1.4 If the Head of Department decides to reduce each learner's mark by 5, determine the effect this will have on the standard deviation. (1)

[10]

QUESTION 2

The table below shows the heart rate per minute of 10 Physical Education students before exercising and 5 minutes after exercising with a skipping rope.

Heart rate before exercising (x)	44	80	56	95	75	78	84	69	76	64
Heart rate after exercising (y)	66	85	69	100	81	90	107	76	88	82

2.1 Represent the above information in a scatter plot ON THE GRID provided. (3)

2.2 Calculate an equation for the least squares regression line for this data. (3)

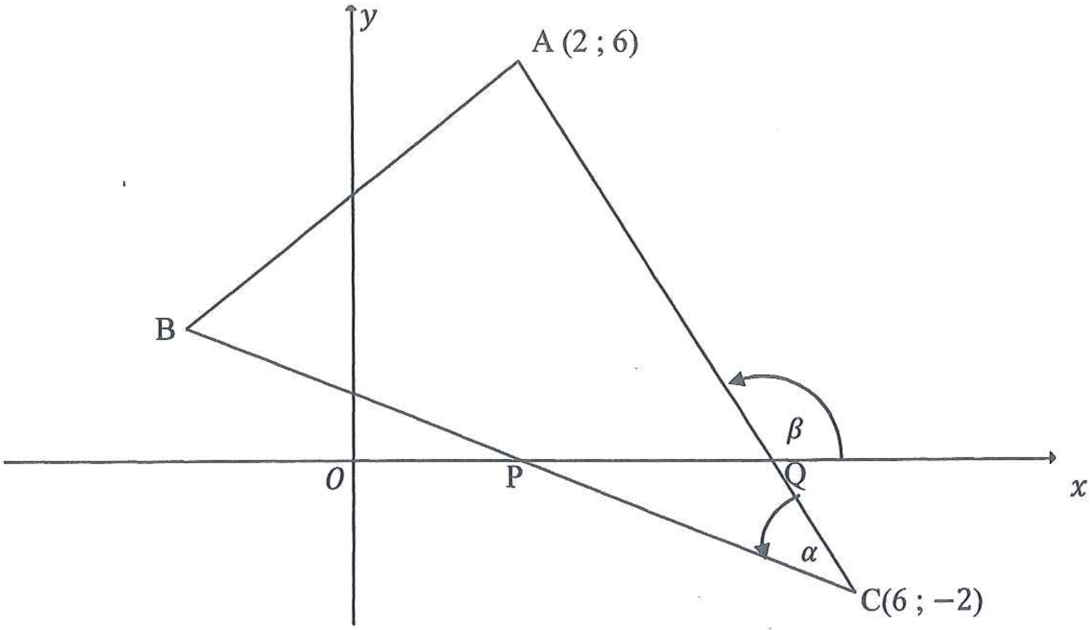
2.3 Draw the least squares regression line on the scatter plot in QUESTION 2.1. (2)

2.4 Calculate the correlation coefficient and comment on the strength of the relationship between the variables. (2)

[10]

QUESTION 3

In the diagram A(2 ; 6) and C(6 ; −1) are vertices of ΔABC and $A\hat{C}B = \alpha$. The angle of elevation of AC is β. The equation of BC is $2y + x = 1$.
The x- intercept of BC is at P and of AC at Q.



3.1 Calculate the:

3.1.1 coordinates of M, the midpoint of AC. (2)

3.1.2 gradient of AC. (2)

3.1.3 equation of the perpendicular bisector of AC. (3)

3.2 Determine whether the point D (6 ; 3) lies on the equation in 3.1.3. (3)

3.3 Determine the equation of the line AC. (2)

3.4 Calculate the size of

3.4.1 β (2)

3.4.2 α (4)

3.5 Calculate the coordinates of P and Q. (3)

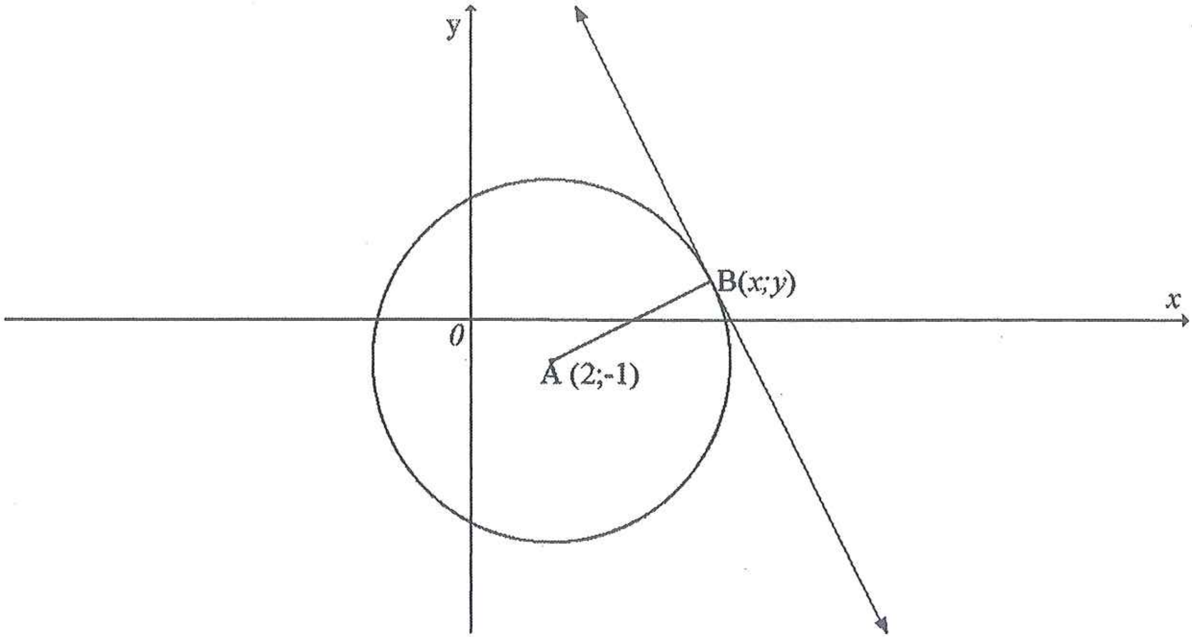
3.6 Hence calculate the area of ΔPQC. (3)

3.7 Determine the equation of the circle, in the form $(x-a)^2 + (y-b)^2 = r^2$ passing through the points A(2 ; 6) and C(6 ; −2) and whose centre lies on the line $y = -2x + 10$. (4)

[28]

QUESTION 4

The line $y = -2x + 13$ is a tangent to the circle with centre A $(2\ ;\ -1)$ at the point B$(x;\ y)$.



4.1 Determine the equation of the line passing through A and B. (3)

4.2 Calculate the coordinates of B. (3)

4.3 Determine the equation of circle A, in the form $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.4 Another circle $x^2 - 16x + y^2 - 4y = -63$ is drawn on the same set of axes. Show with the necessary calculations that the two circles touch externally. (5)

[14]

QUESTION 5

5.1 If $\sin 37° = p$, express the following in terms of p:

5.1.1 $\sin 577°$ (2)

5.1.2 $\cos 16°$ (4)

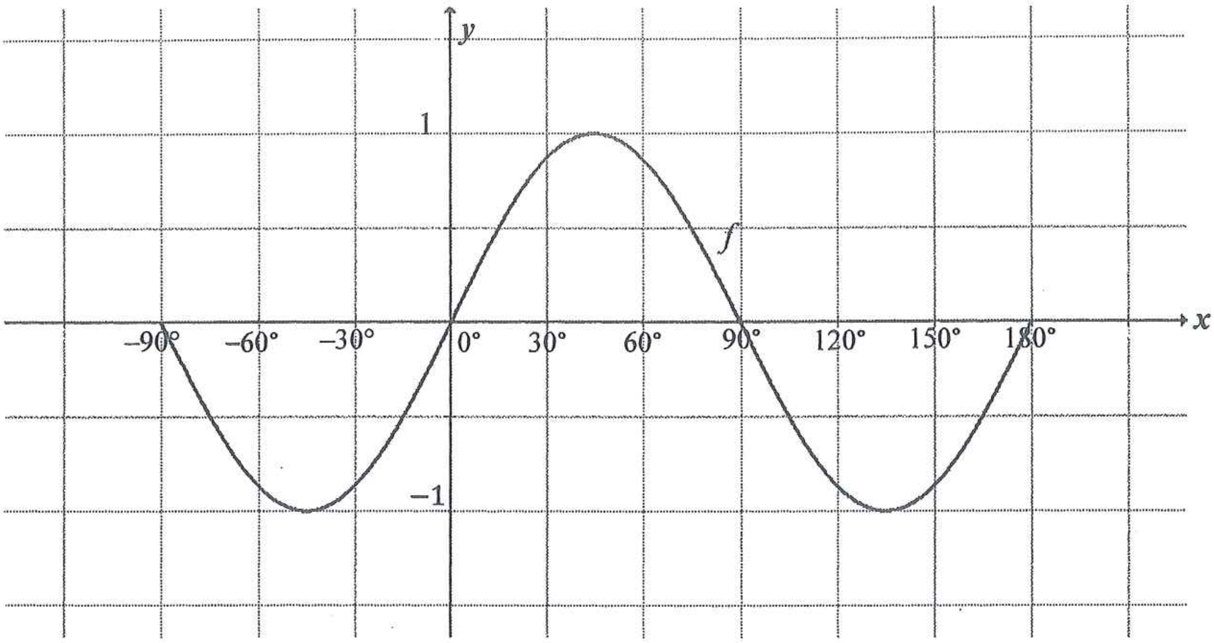
5.2 5.2.1 Simplify $\dfrac{6\sin(180° - x).\cos(x - 360°)}{\sin^2 x - \sin^2(90° - x)}$ to one trigonometric ratio. (6)

5.2.2 Hence, without using a calculator, calculate $\dfrac{6\sin 15°.\cos 15°}{\sin^2 15° - \cos^2 15°}$ (2)

[14]

QUESTION 6

The graph of $f(x) = \sin 2x$ is drawn in the diagram for the interval $x \in [-90°;180°]$



6.1 Draw on the same set of axes the graph of $g(x) = \cos(x + 60°)$ for $x \in [-90°\,;180°]$. Clearly show all intercepts with the axes, coordinates of the turning points and the endpoints of the graphs. (3)

6.2 Calculate the general solution for $\sin 2x = \cos(x + 60°)$. (6)

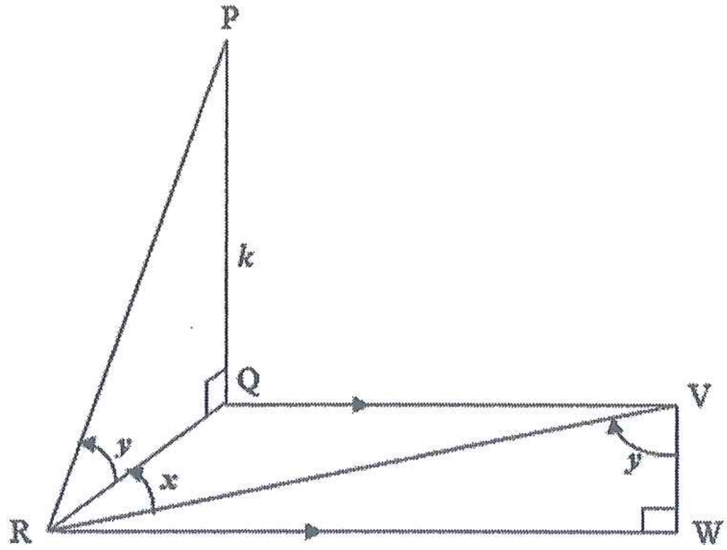
6.3 Write down the solution for $\sin 2x = \cos(x + 60°)$, $x \in [-90°\,;180°]$. (2)

6.4 Determine for which values of x is $f(x).g(x) < 0$, for $x \in [-90°\,;180°]$. (3)

[14]

QUESTION 7

In the diagram, QVWR is a sports field in a horizontal plane with QV || RW and VW ⊥ RW. PQ is a vertical pylon for a floodlight such that PQ is k units. The angle of elevation of P from R is y. $Q\hat{R}V = x$ and $R\hat{V}W = y$



7.1 Show that $R\hat{Q}V = 90° - x + y$ (3)

7.2 Express RQ in terms of y and k. (2)

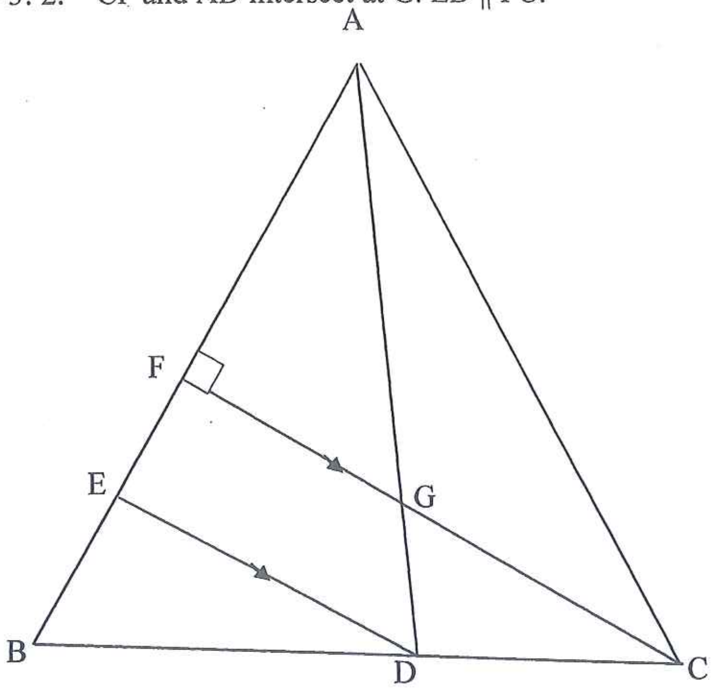
7.3 Hence prove that $VR = \dfrac{k\cos(x-y)}{\sin y}$ (4)

[9]

Give reasons for ALL statements in QUESTIONS 8, 9 and 10

QUESTION 8

In the diagram, CF is the perpendicular bisector of AB in ΔABC. D is a point on BC such that BD : DC = 3: 2. CF and AD intersect at G. ED || FC.



Determine, giving reasons:

8.1 $\frac{AF}{AE}$ (4)

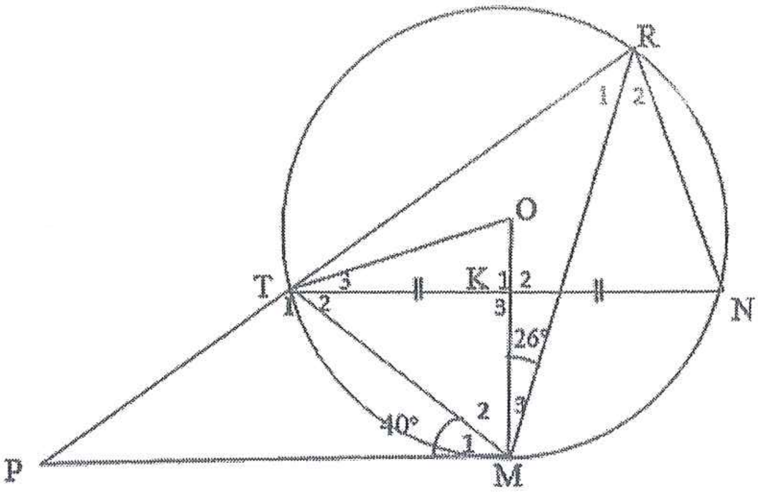
8.2 $\frac{DE}{FG}$ (4)

8.3 $\frac{\text{area of } \Delta GFA}{\text{area of } \Delta DEA}$ (3)

[11]

QUESTION 9

In the diagram below, O is the centre of circle TRNM. MP is a tangent to the circle at M such that RT produced meets MP at P. OM intersects TN at K. K is the midpoint of TN.
$P\hat{M}T = 40°$ and $\hat{M}_3 = 26°$.



Calculate, with reasons, the size of:

9.1 $T\hat{O}M$ (4)

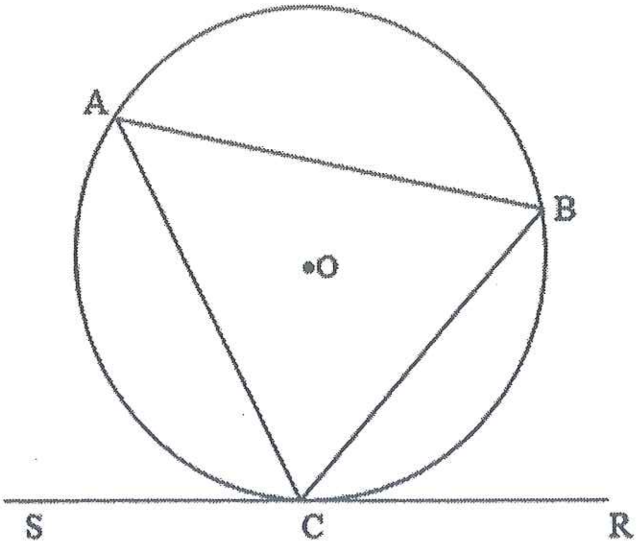
9.2 $\hat{N}$ (4)

9.3 $\hat{T}_3$ (3)

[11]

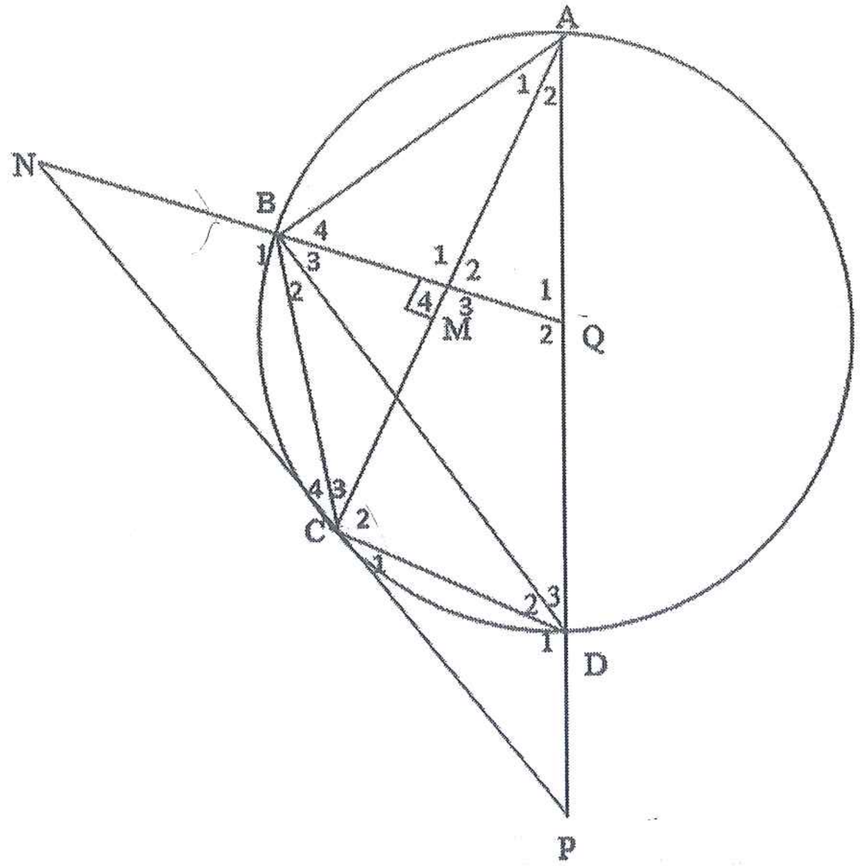
QUESTION 10

10.1 In the diagram below the circle with centre O passes through the points A, B and C. SCR is a tangent to the circle at C. AC, AB and BC are joined.



Use the diagram to prove the theorem that states that $B\hat{C}R = \hat{A}$ (6)

10.2 In the diagram AD is a diameter of the circle ABCD. NCP is a tangent to the circle at C. AD produced, meets tangent NCP at P and QB produced meets NCP at N. AC intersects NQ such that AC ⊥ NQ.



10.2.1 Prove that $NQ \parallel CD$. (3)

10.2.2 Prove that ANCQ is a cyclic quadrilateral. (4)

10.2.3 (a) Prove that $\Delta PCD \parallel\mid \Delta PAC$ (3)

(b) Hence complete the statement: $PC^2 = \ldots$ (2)

10.2.4 (a) Prove $\Delta NBC \parallel\mid \Delta BCD$ (4)

(b) Hence show that $BC^2 = CD.NB$ (2)

10.2.5 If it is given that $PC = MC$, prove that $1 - \frac{BM^2}{BC^2} = \frac{AP.DP}{CD.NB}$ (5)

[29]

TOTAL: 150

INFORMATION SHEET: MATHEMATICS

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $A = P(1+ni)$ $A = P(1-ni)$ $A = P(1-i)^n$ $A = P(1+i)^n$

$\sum_{i=1}^{n} 1 = n$ $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$ $T_n = a + (n-1)d$ $S_n = \frac{n}{2}(2a + (n-1)d)$

$T_n = ar^{n-1}$ $S_n = \frac{a(r^n - 1)}{r-1}$; $r \neq 1$ $S_\infty = \frac{a}{1-r}$; $-1 < r < 1$

$F = \frac{x[(1+i)^n - 1]}{i}$ $P = \frac{x[1-(1+i)^{-n}]}{i}$ $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ $M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right)$ $y = mx + c$

$y - y_1 = m(x - x_1)$ $m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \tan\theta$ $(x-a)^2 + (y-b)^2 = r^2$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $a^2 = b^2 + c^2 - 2bc.\cos A$

$area\ \Delta ABC = \frac{1}{2}ab.\sin C$

$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta$ $\sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta$ $\cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$

$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases}$ $\sin 2\alpha = 2\sin\alpha.\cos\alpha$

$(x ; y) \to (x\cos\theta + y\sin\theta ; y\cos\theta - x\sin\theta)$

$(x ; y) \to (x\cos\theta - y\sin\theta ; y\cos\theta + x\sin\theta)$

$\bar{x} = \frac{\sum fx}{n}$ $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$

$P(A) = \frac{n(A)}{n(S)}$ $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$\hat{y} = a + bx$ $b = \frac{\sum(x - \bar{x})(y - \bar{y})}{\sum(x - \bar{x})^2}$

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DEPARTMENT OF
EDUCATION

NASIONALE SENIOR SERTIFIKAAT

GRAAD 12

WISKUNDE VRAESTEL 2

SEPTEMBER 2017

PUNTE: 150

TYD: 3 uur

Hierdie vraestel bestaan uit 11 bladsye, 'n inligtingsblad en 'n 15-bladsy antwoordboek.

INSTRUKSIES EN INLIGTING

Lees die volgende instruksies aandagtig deur voordat die vrae beantwoord word.

1. Hierdie vraestel bestaan uit 10 vrae.
2. Beantwoord AL die vrae in die ANTWOORDBOEK wat verskaf word.
3. Dui ALLE berekenings, diagramme, grafieke, ensovoorts wat jy in die bepaling van jou antwoorde gebruik het, duidelik aan.
4. Volpunte sal nie noodwendig aan antwoorde alleen toegeken word nie.
5. Jy mag 'n goedgekeurde, wetenskaplike sakrekenaar (nie-programmeerbaar en nie-grafies) gebruik, tensy anders vermeld.
6. Rond antwoorde, indien nodig, tot TWEE desimale plekke af, tensy anders vermeld.
7. Diagramme is NIE noodwendig volgens skaal geteken NIE.
8. 'n Inligtingsblad met formules, is aan die einde van die vraestel ingesluit.
9. Skryf leesbaar en bied jou werk netjies aan.

VRAAG 1

'n Klas met 25 leerders het 'n toets geskryf wat uit 25 tel. Die punte is as volg:

PUNTE BEHAAL	FREKWENSIE
8	2
10	3
12	4
17	5
20	5
22	4
23	2

1.1 Bereken die gemiddelde van die data. (2)

1.2 Bepaal die 1.2.1 mediaan (2)

1.2.2 standaardafwyking van die data. (2)

1.3 Bepaal hoeveel van die leerders se punte sal binne EEN standaardafwyking van die gemiddeld wees. Toon al die nodige bewerkings. (3)

1.4 Die Departementshoof besluit om elke leerder se punt met 5 punte te verminder. Bepaal watse effek dit op die standaardafwyking sal hê. (1)

[10]

VRAAG 2

Die tabel hieronder wys die polsslag per minuut van 10 Lewensoriëntering leerders voor hulle met 'n springtou oefen en 5 minute na hulle met 'n springtou geoefen het.

Polsslag voor oefening (x)	44	80	56	95	75	78	84	69	76	64
Polsslag na oefening (y)	66	85	69	100	81	90	107	76	88	82

2.1 Stel die inligting voor in 'n spreidiagram OP DIE ASSESTEL hierna. (3)

2.2 Bereken die vergelyking van die kleinstekwadrate - regressielyn van die data. (3)

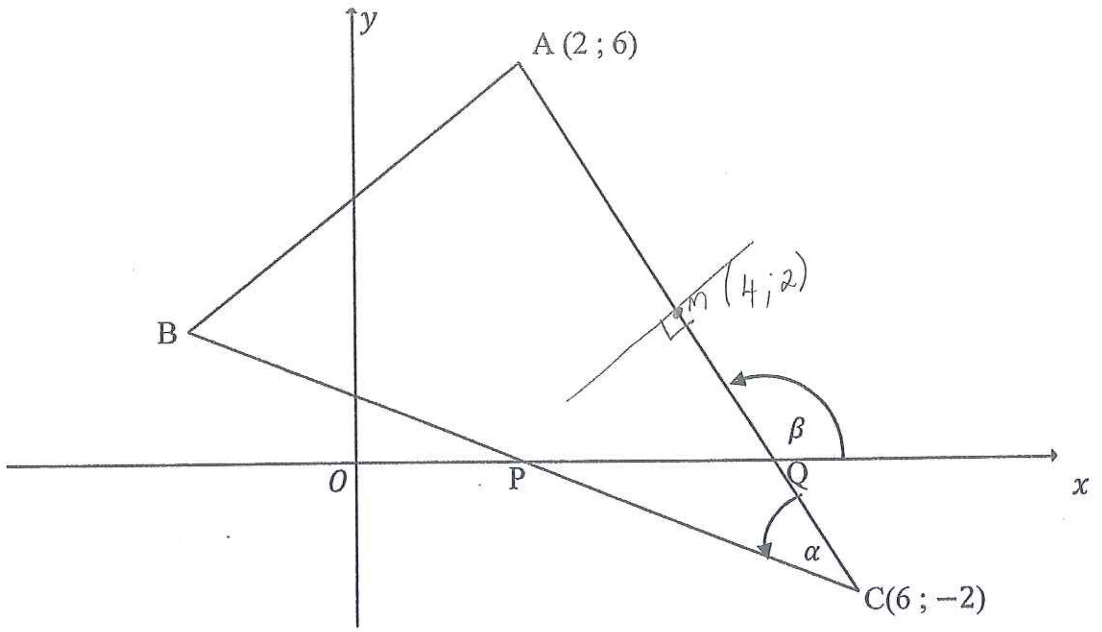
2.3 Teken die kleinstekwadrate - regressielyn op die diagram in VRAAG 2.1. (2)

2.4 Bereken die korrelasiekoëffisiënt en lewer kommentaar oor die sterkte van die verwantskap tussen die veranderlikes. (2)

[10]

VRAAG 3

In die diagram is A(2 ; 6) en C(6 ; −1) hoekpunte van ΔABC en $A\hat{C}B = \alpha$. Die inklinasiehoek van AC is β. Die vergelyking van BC is $2y + x = 1$.
Die x- afsnit van BC is by P en van AC by Q.



3.1 Bereken die:

3.1.1 koördinate van M, die middelpunt van AC. (2)

3.1.2 gradiënt van AC. (2)

3.1.3 vergelyking van die middelloodlyn van AC. (3)

3.2 Bepaal of die punt D (6 ; 3) op die vergelyking in 3.1.3 lê. (3)

3.3 Bepaal die vergelyking van die lyn AC. (2)

3.4 Bereken die grootte van:

3.4.1 β (2)

3.4.2 α (4)

3.5 Bereken die koördinate van P en Q. (3)

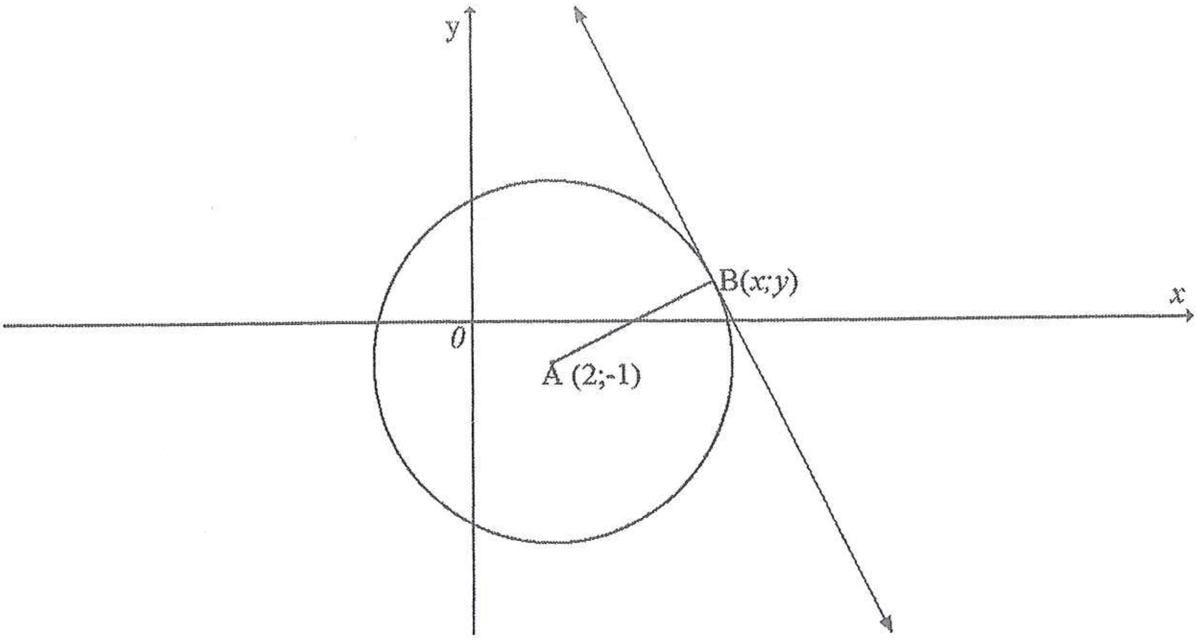
3.6 Bereken vervolgens die oppervlakte van ΔPQC. (3)

3.7 Bepaal die vergelyking van die sirkel, in die vorm $(x-a)^2 + (y-b)^2 = r^2$, wat deur die punte A(2 ; 6) en C(6 ; −2) gaan en waarvan die middelpunt op die lyn $y = -2x + 10$ lê. (4)

[28]

VRAAG 4

Die lyn $y = -2x + 13$ is 'n raaklyn aan die sirkel met middelpunt A $(2\,;\,-1)$ by die punt B$(x;\ y)$.



4.1 Bepaal die vergelyking van die lyn wat deur A en B gaan. (3)

4.2 Bereken die koördinate van B. (3)

4.3 Bereken die vergelyking van sirkel A, in die vorm $(x-a)^2 + (y-b)^2 = r^2$. (3)

4.4 'n Ander sirkel $x^2 - 16x + y^2 - 4y = -63$ word op dieselfde assestel geteken. Toon aan, met die nodige bewerkings, dat die twee sirkels uitwendig raak. (5)

[14]

VRAAG 5

5.1 As $\sin 37° = p$, druk die volgende uit in terme van p:

5.1.1 $\sin 577°$ (2)

5.1.2 $\cos 16°$ (4)

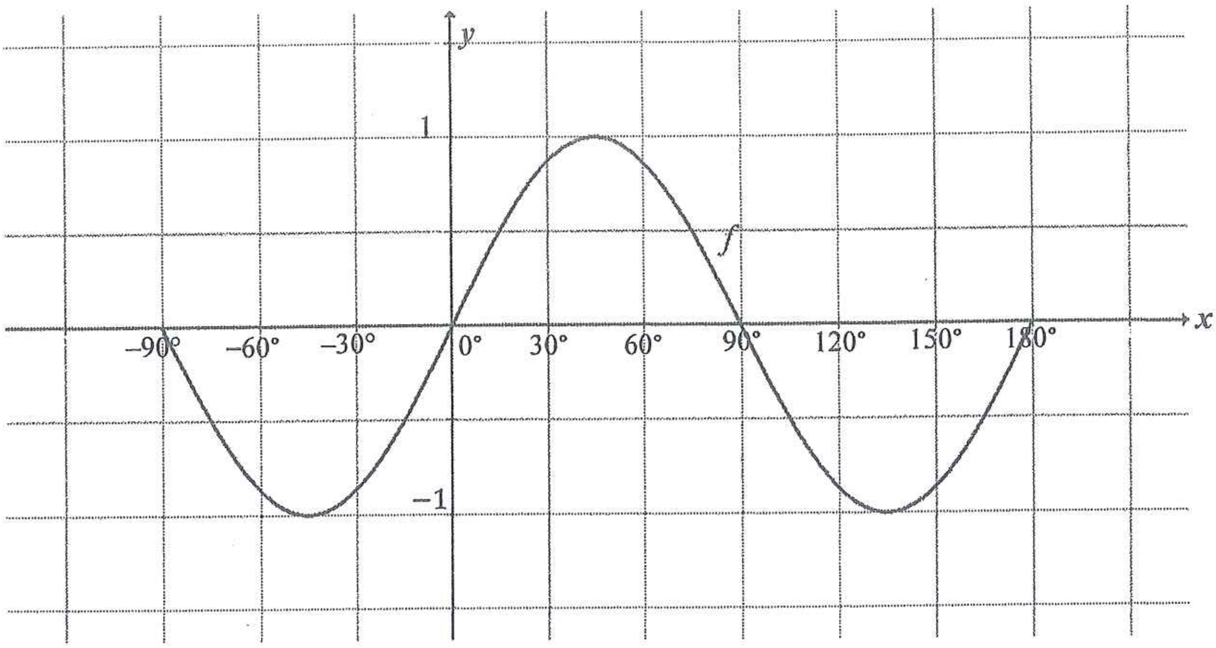
5.2 5.2.1 Vereenvoudig $\dfrac{6\sin(180° - x).\cos(x - 360°)}{\sin^2 x - \sin^2(90° - x)}$ tot een trig verhouding. (6)

5.2.2 Bereken vervolgens, sonder 'n sakrekenaar, $\dfrac{6\sin 15°.\cos 15°}{\sin^2 15° - \cos^2 15°}$ (2)

[14]

VRAAG 6

Die grafiek van $f(x) = \sin 2x$ is geteken in die diagram vir die interval $x \in [-90°;180°]$.



6.1 Teken op dieselfde assestel die grafiek van $g(x) = \cos(x + 60°)$ vir $x \in [-90° ; 180°]$. Dui al die afsnitte met die asse, die koördinate van die draaipunte en die eindpunte van die grafiek duidelik aan. (3)

6.2 Bereken die algemene oplossing vir $\sin 2x = \cos(x + 60°)$. (6)

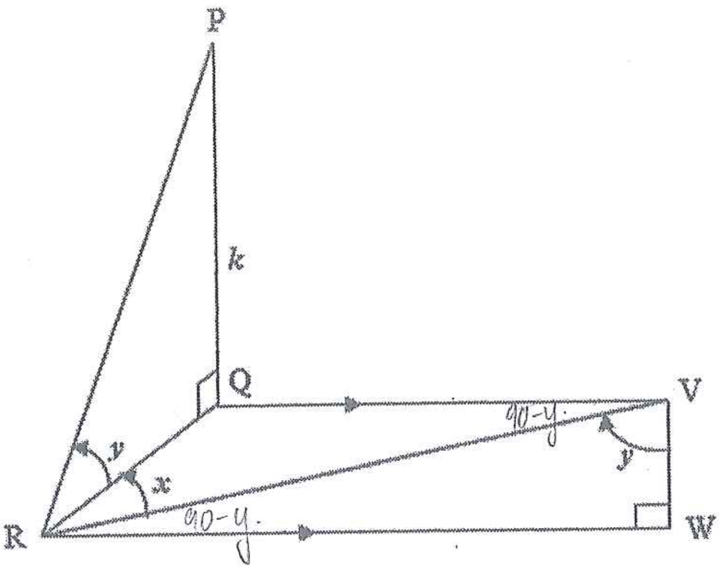
6.3 Skryf die oplossing neer vir $\sin 2x = \cos(x + 60°)$, $x \in [-90° ; 180°]$. (2)

6.4 Bepaal vir watter waardes van x is $f(x).g(x) < 0$, vir $x \in [-90° ; 180°]$. (3)

[14]

VRAAG 7

In die diagram is QVWR 'n sportveld in 'n horisontale vlak met QV || RW en VW ⊥ RW. PQ is 'n vertikale toring vir 'n spreilig sodat die lengte van PQ k eenhede is. Die hoogtehoek van P vanaf R is y. $Q\hat{R}V = x$ en $R\hat{V}W = y$



7.1 Toon aan dat $R\hat{Q}V = 90° - x + y$ (3)

7.2 Druk RQ uit in terme van y en k. (2)

7.3 Bewys vervolgens dat $VR = \dfrac{k\cos(x-y)}{\sin y}$ (4)

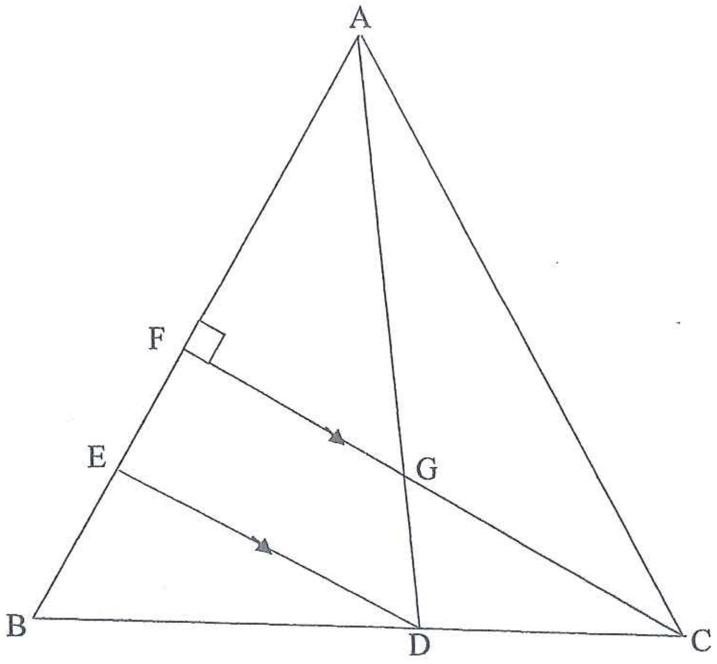
[9]

Gee redes vir AL die bewerings in VRAE 8, 9 en 10

VRAAG 8

In die diagram is CF die middelloodlyn van AB in ΔABC. D is 'n punt op BC sodat

$BD : DC = 3 : 2$. CF en AD sny by G. $ED \parallel FC$.



Bepaal, met redes:

8.1 $\dfrac{AF}{AE}$ (4)

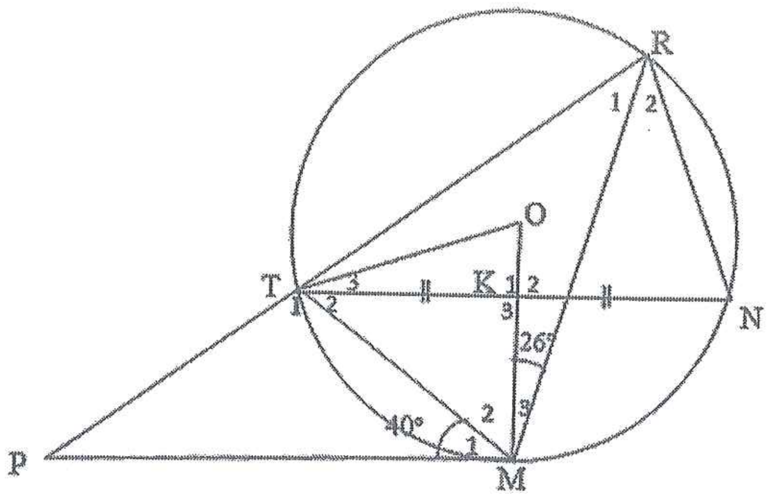
8.2 $\dfrac{DE}{FG}$ (4)

8.3 $\dfrac{\text{oppervlak van } \Delta GFA}{\text{oppervlak van } \Delta DEA}$ (3)

[11]

VRAAG 9

In die diagram is O die middelpunt van sirkel TRNM. MP is 'n raaklyn aan die sirkel by M, sodanig dat as RT verleng word, dit MP by P sny. OM sny TN by K. K is die middelpunt van TN. $P\hat{M}T = 40°$ en $\hat{M}_3 = 26°$.



Bereken, met redes, die grootte van:

9.1 $T\hat{O}M$ (4)

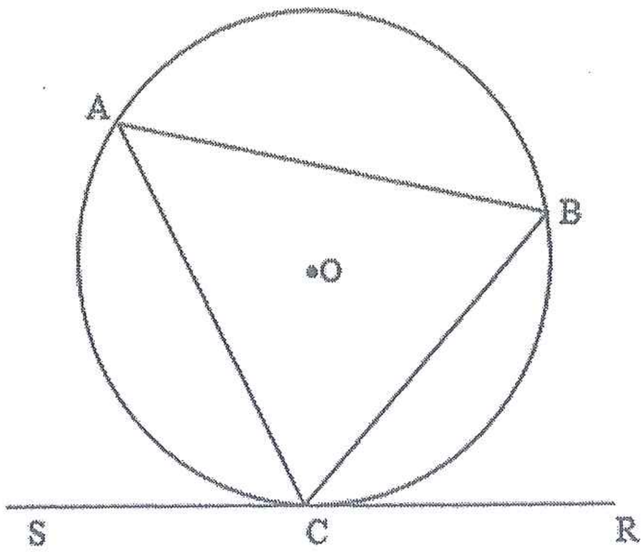
9.2 $\hat{N}$ (4)

9.3 $\hat{T}_3$ (3)

[11]

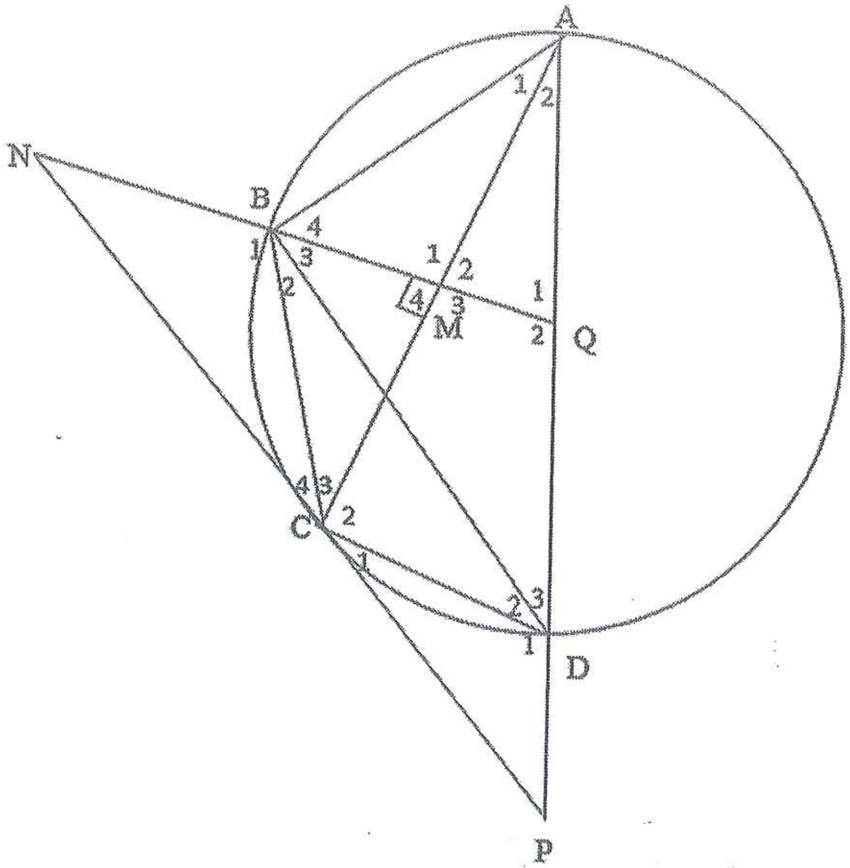
VRAAG 10

10.1 In die diagram gaan die sirkel met middelpunt O deur die punte A, B en C. SCR is 'n raaklyn aan die sirkel by C. AC, AB en BC word verbind.



Gebruik die diagram en bewys die stelling wat beweer dat $B\hat{C}R = \hat{A}$. (6)

10.2 In die diagram is AD 'n middellyn van sirkel ABCD. NCP is 'n raaklyn aan die sirkel by C. As AD verleng word, sny dit raaklyn NCP by P en QB verleng, sny NCP by N. AC sny NQ so dat AC ⊥ NQ.



10.2.1 Bewys dat $NQ \parallel CD$. (3)

10.2.2 Bewys dat ANCQ 'n koordevierhoek is. (4)

10.2.3 (a) Bewys dat $\Delta PCD \,|||\, \Delta PAC$ (3)

(b) Voltooi vervolgens die bewering: $PC^2 = \ldots$ (2)

10.2.4 (a) Bewys dat $\Delta NBC \,|||\, \Delta BCD$ (4)

(b) Toon vervolgens aan dat $BC^2 = CD.NB$ (2)

10.2.5 As dit verder gegee word dat $PC = MC$, bewys dat $1 - \frac{BM^2}{BC^2} = \frac{AP.DP}{CD.NB}$ (5)

[29]

TOTAAL: 150

INLIGTINGSBLAD: WISKUNDE

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad A = P(1+ni) \quad A = P(1-ni) \quad A = P(1-i)^n \quad A = P(1+i)^n$$

$$\sum_{i=1}^{n} 1 = n \quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2} \quad T_n = a + (n-1)d \quad S_n = \frac{n}{2}(2a + (n-1)d)$$

$$T_n = ar^{n-1} \quad S_n = \frac{a(r^n - 1)}{r-1} \; ; \quad r \neq 1 \quad S_\infty = \frac{a}{1-r}; \; -1 < r < 1$$

$$F = \frac{x[(1+i)^n - 1]}{i} \quad P = \frac{x[1-(1+i)^{-n}]}{i} \quad f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad M\left(\frac{x_1 + x_2}{2}; \frac{y_1 + y_2}{2}\right) \quad y = mx + c$$

$$y - y_1 = m(x - x_1) \quad m = \frac{y_2 - y_1}{x_2 - x_1} \quad m = \tan\theta \quad (x-a)^2 + (y-b)^2 = r^2$$

In ΔABC: $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $\quad a^2 = b^2 + c^2 - 2bc.\cos A$

$$area\, \Delta ABC = \frac{1}{2} ab.\sin C$$

$$\sin(\alpha + \beta) = \sin\alpha.\cos\beta + \cos\alpha.\sin\beta \quad \sin(\alpha - \beta) = \sin\alpha.\cos\beta - \cos\alpha.\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha.\cos\beta - \sin\alpha.\sin\beta \quad \cos(\alpha - \beta) = \cos\alpha.\cos\beta + \sin\alpha.\sin\beta$$

$$\cos 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 1 - 2\sin^2\alpha \\ 2\cos^2\alpha - 1 \end{cases} \quad \sin 2\alpha = 2\sin\alpha.\cos\alpha$$

$$(x\,;\,y) \rightarrow (x\cos\theta + y\sin\theta\,;\, y\cos\theta - x\sin\theta)$$
$$(x\,;\,y) \rightarrow (x\cos\theta - y\sin\theta\,;\, y\cos\theta + x\sin\theta)$$

$$\bar{x} = \frac{\sum fx}{n} \quad \sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}$$

$$P(A) = \frac{n(A)}{n(S)} \quad P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$\hat{y} = a + bx \quad b = \frac{\sum(x-\bar{x})(y-\bar{y})}{\sum(x-\bar{x})^2}$$