education

DEPARTMENT: EDUCATION
MPUMALANGA PROVINCE

NATIONAL SENIOR CERTIFICATE MEMORANDUM

MATHEMATICS MEMORANDUM P2

SEPTEMBER 2017

GRADE 12

MARKS: 150

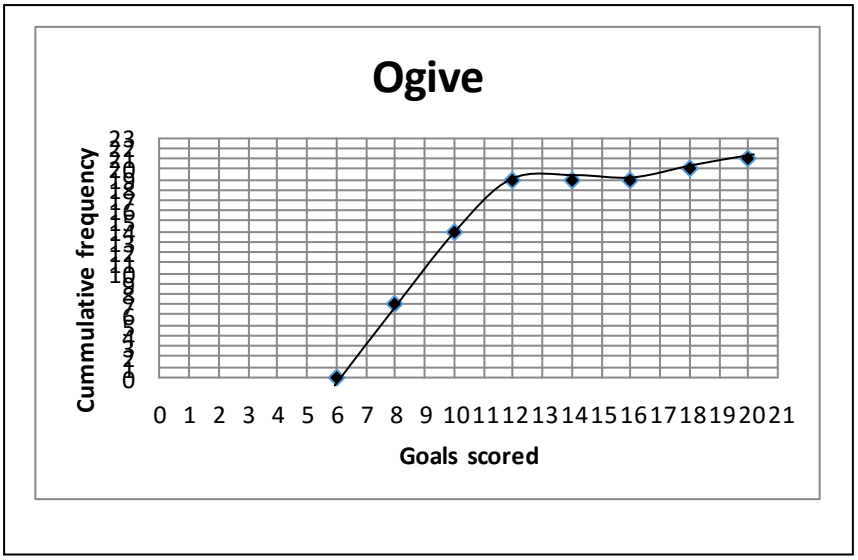
TIME: 3 Hours

This memo consists of 19 pages.

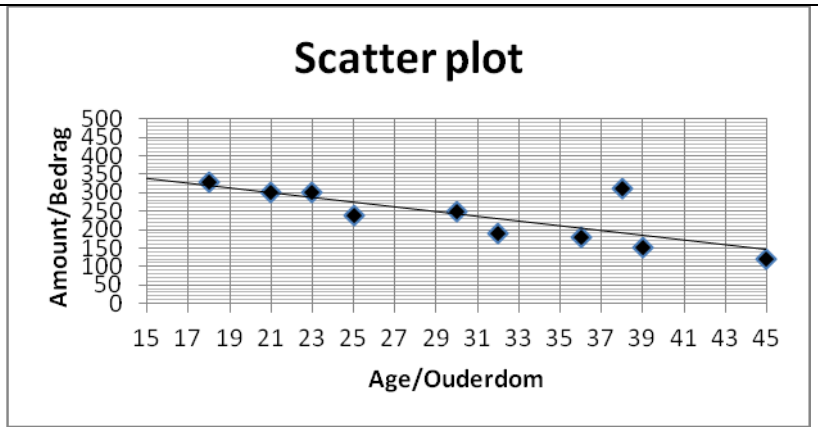
QUESTION 1

	Solution	Mark
1.1	$p = 7$	✓ answer (1)
1.2	$\frac{(7\times 7)+(9\times 7)+(11\times 5)+(17\times 1)+(19\times 1)}{21} = \frac{203}{21} = \frac{29}{3} = 9{,}67$	✓calculation of mean ✓ answer (2)
1.3		

Goals scored	Frequency	Cumulative frequency	
$6 < x \leq 8$	7	7	✓✓ Cum. Freq. (2)
$8 < x \leq 10$	p	**14**	
$10 < x \leq 12$	5	19	
$12 < x \leq 14$	0	19	
$14 < x \leq 16$	0	19	
$16 < x \leq 18$	1	20	
$18 < x \leq 20$	1	21	
TOTAL	21		

1.4		✓ upper limit ✓ cumulative frequency ✓ shape ✓ grounding (4)
1.5	$IQR = Q_3 - Q_1$ $= 11 - 7{,}3$ $= 3{,}7$	✓ Q_3 ✓ Q_1 (accept $7{,}1 \leq Q_1 \leq 7{,}5$) ✓ Answer (3)
		[12]

QUESTION 2

	Solution	Mark
2.1	$A = 434.47$ $B = -6,43$ $y = 434,47 - 6,43x$	✓ $A = 434.47$ ✓ $B = -6,43$ ✓equation (3)
2.2	$y = 434,47 - 6,43x$ $y = 434,47 - 6,43(27)$ $= R260,86$	✓ substitution ✓answer (2)
2.3		✓ y- intercept (accept $430 \leq y \leq 440$) ✓ Line (2)
2.4	$r = -0,772557$	✓✓ answer (2)
2.5	Strongly negative	✓ strong (1)
		[10]

QUESTION 3

3.1	Solution	Mark
3.1.1	(a) $G\left(\frac{-2+4}{2};\frac{-3-5}{2}\right)$ $G(1;-4)$	✓ x-value ✓ y-value (2)
	(b) $BG=\sqrt{(1-4)^2+(-4+5)^2}$ $=\sqrt{10}$ $AB=2BG$ $=2\sqrt{10}$ **OR** $AB=\sqrt{(-2-4)^2+(-3+5)^2}$ $=\sqrt{40}$ $=2\sqrt{10}$	✓ substitution ✓ answer (2) ✓ substitution ✓ answer (2)
3.1.2	$BG=\sqrt{(1-4)^2+(-4+5)^2}$ $=\sqrt{10}$ $(x-1)^2+(y-(-4))^2=\left(\sqrt{10}\right)^2$ $(x-1)^2+(y+4)^2=10$	✓ substitution ✓ answer ✓ correct subst. ✓ answer (4)
3.1.3	BG = GE $=\sqrt{10}$ (radii)	✓ answer (1)
3.1.4	$M_{AC}\left(\frac{-2+2}{2};\frac{-3+1}{2}\right)$ $=M_{AC}(0;-1)$ $x_D=\frac{x+4}{2}=0$ $y_D=\frac{y-5}{2}=-1$ $x_D=-4$ $y_D=3$ $D(-4;3)$	✓ $M_{AC}(0;-1)$ ✓ $y_D=3$ ✓ $x_D=-4$ ✓ answer (4)

3.2.1	$m_{QR} = 0$ **OR** $p - 7 = -1$ $\frac{p-7+1}{4p-4} = 0$ $p = 6$ $p = 6$	✓method ✓ answer (2)
3.2.2	$m_{TP} = m_{PQ}$ $\frac{3-(t+2)}{-2-(2t-11)} = \frac{-1-3}{4+2}$ $\frac{3-t-2}{-2-2t+11} = \frac{-2}{3}$ $\frac{-t+1}{-2t+9} = \frac{-2}{3}$ $3(-t+1) = -2(-2t+9)$ $-3t+3 = 4t-18$ $21 = 7t$ $t = 3$ **OR** $m_{TQ} = m_{PQ}$ $\frac{t+2-(-1)}{2t-11-4} = \frac{-1-3}{4+2}$ $\frac{t+3}{2t-15} = \frac{-2}{3}$ $3t+9 = -4t+30$ $21 = 7t$ $t = 3$	✓equating gradients ✓ subst. ✓ simplification ✓removing fraction ✓ answer (5) ✓equating gradients ✓ subst. ✓ simplification ✓removing fraction ✓ answer (5)
		[20]

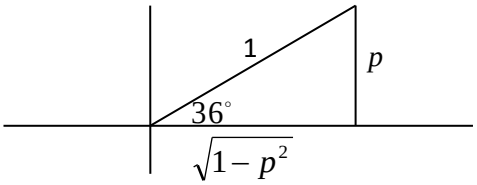
QUESTION 4

	Solution	Mark
4.1.1	$x^2 + y^2 - 2x + 10y = 35$ $x^2 - 2x + 1 + y^2 + 10y + 25 = 35 + 1 + 25$ $(x-1)^2 + (y+5)^2 = 61$ $\text{P}(1;-5)$	✓ completing the square ✓ std form ✓ 61 ✓ answer (4)
4.1.2	$r = \sqrt{61}$	✓ answer (1)
4.2	$\text{PH} \perp \text{AB}$ radius and tangent $m_1 \times m_2 = -1$ $m_{PH} = \frac{1}{2}$ $y + 5 = \frac{1}{2}(x-1)$ $y = \frac{1}{2}x - \frac{11}{2}$	✓ $m_1 \times m_2 = -1$ ✓ $m_{PH} = \frac{1}{2}$ subst $(1;-5)$ ✓ equation of PH (4)
4.3	$-2x + 7 = \frac{1}{2}x - \frac{11}{2}$ $-4x + 14 = x - 11$ $-5x = -25$ $y = -2(5) + 7$ $x = 5$ $y = -3$ $\therefore \quad H(5;-3)$	✓ equating ✓ $-5x = -25$ ✓ $y = -2(5) + 7$ ✓answer (4)
4.4	$\text{AP}^2 = \text{PH}^2 + \text{AH}^2$ $\left(\sqrt{61}\right)^2 = \left(\sqrt{20}\right)^2 + \text{AH}^2$ $\sqrt{41} = \text{AH}$ $\therefore \quad \text{AB} = 2\sqrt{41}$ radius and chord **OR**	✓ subst ✓ length of AH ✓ length of AB (3)

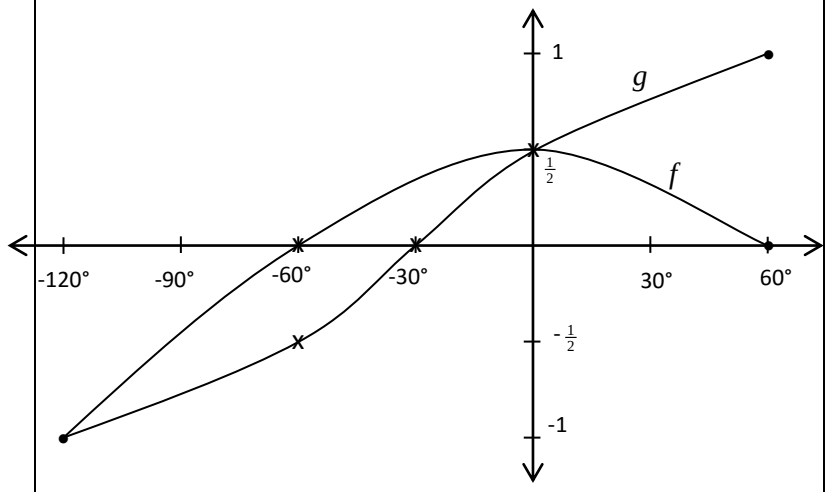
	In ΔAPH: $\cos\hat{P} = \frac{PH}{PA}$ $= 0,5725$ $\hat{P} = 55,1°$ $\frac{AH}{\sin\hat{P}} = \frac{AP}{\sin 90°}$ $AH = \frac{\sin 55,1°.\sqrt{61}}{\sin 90°}$ $= 6,405$ AB = 6,41	✓ $\hat{P} = 55,1°$ ✓ ratio ✓ answer (3)
4.5	$\tan\alpha = -2$ $\alpha = 180° - 6.343°$ $\alpha = 116,57°$ $\theta = 180° - 116,57°$ opp $\angle s$ of cyclic quad $= 63,43°$	✓ $\tan\alpha = -2$ ✓ $\alpha = 116,57°$ ✓ $\theta = 180° - 116,57°$ ✓ answer (4)
		[20]

QUESTION 5

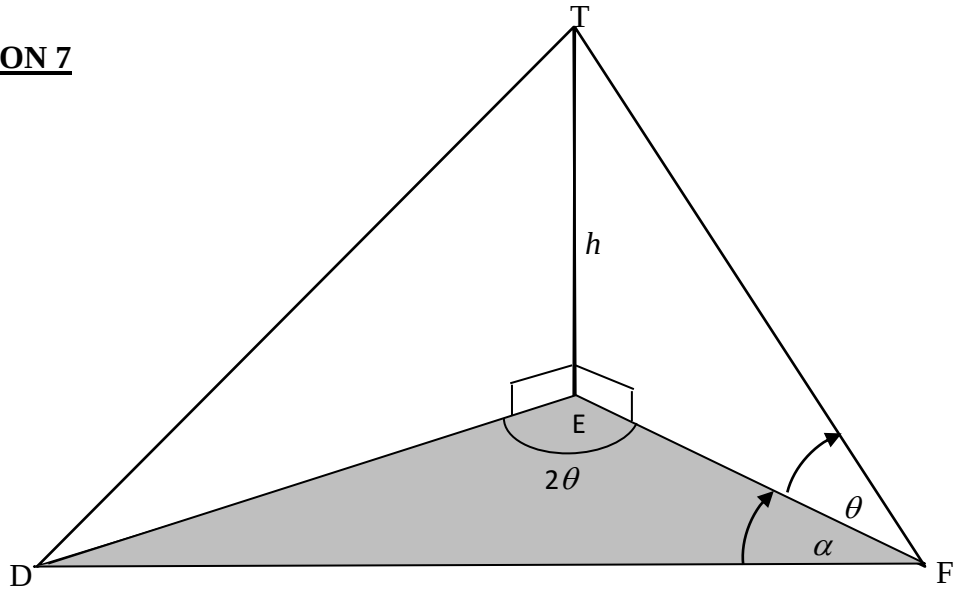
	Solution	Mark
5.1	$\cos(x+65^\circ).\cos(x+20^\circ)-\sin(x+245^\circ).\sin(x+20^\circ)$ $=\cos(x+65^\circ).\cos(x+20^\circ)+\sin(x+65^\circ).\sin(x+20^\circ)$ $=\cos[(x+65^\circ)-(x+20^\circ)]$ $=\cos 45^\circ$ $=\frac{\sqrt{2}}{2}$	✓ reduction ✓ compound angle rule ✓simplification ✓answer (4)
5.2	$\sin x=\cos 2x \quad$ for $\quad x\in[-180^\circ;180^\circ]$ $\sin x=1-2\sin^2 x$ $2\sin^2+\sin x-1=0$ $(2\sin x-1)(\sin x+1)=0$ $\sin x=\frac{1}{2}$ or $\sin x=-1$ $x=30^\circ+k.360^\circ;\ k\in Z$ or $x=150^\circ+k.360^\circ;\ k\in Z$ $x=-90^\circ+k.360^\circ;\ k\in Z$ or $x=270^\circ+k.360^\circ, k\in Z$ $\therefore \quad x\in\{-90^\circ;30^\circ;150^\circ\}$ **OR** $\cos(90^\circ-x)=\cos 2x$ $90^\circ-x=2x+k.360^\circ;\quad k\in Z$ $-3x=-90^\circ+k.360^\circ;\ k\in Z$ $x=30^\circ+k.120^\circ;\ k\in Z$ $90^\circ-x=360^\circ-2x+k.360^\circ;\ k\in Z$ $x=270^\circ+k.360^\circ;\ k\in Z$ $x\in\{-90^\circ;30^\circ;150^\circ\}$	✓ compound angle rule ✓ std form ✓ factors ✓ ref angle ✓✓ answers (6) ✓ co-function ✓ $90^\circ-x=2x+k.360^\circ$ ✓ $x=30^\circ+k.120^\circ$ ✓ $x=270^\circ+k.360^\circ$ ✓✓ answers (6)

5.3.1	$\cos 36^\circ = \frac{\sqrt{1-p^2}}{1}$ 	✓Pythagoras ✓ answer (2)
5.3.2	$\cos 72^\circ$ $\cos 2(36^\circ)$ $= 1 - 2\sin^2 36^\circ$ $= 1 - 2p^2$	✓ double angle ✓ expansion ✓ answer (3)
5.3.3	$= \sin(36^\circ + 30^\circ)$ $= \sin 36^\circ.\cos 30^\circ + \cos 36^\circ.\sin 30^\circ$ $= p.\frac{\sqrt{3}}{2} + \sqrt{1-p^2}.\frac{1}{2}$	✓ compound angle ✓ expansion ✓ answer (3)
		[18]

QUESTION 6 **[13]**

6.1		graph of f : ✓ x- intercepts ✓ y- intercept ✓ turning point graph of g : ✓ x- intercept ✓ y- intercept ✓ endpoints (6)
6.2.1	$f(x) > g(x)$ $x \in (-120^\circ; 0^\circ)$	✓ notation ✓ values (2)
6.2.2	$x = -30^\circ$	✓ answer (1)
6.2.3	$\cos(60^\circ - x) < 0$ $\sin[90^\circ - (60^\circ - x)] < 0$ $\sin(30^\circ + x) < 0$ $x \in [-120^\circ; -30^\circ)$	✓ co-function ✓ $\sin(30^\circ + x) < 0$ ✓ notation ✓ endpoints (4)
		[13]

QUESTION 7

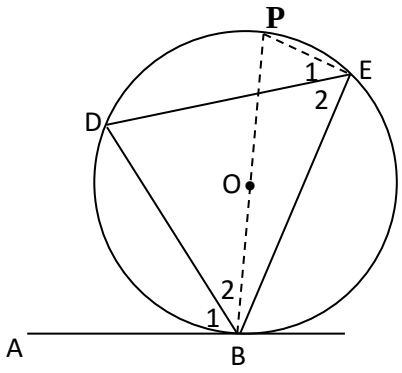


7.1	$\frac{h}{EF} = \tan\theta$ $\text{EF} = \frac{h}{\tan\theta}$	✓ ratio ✓ answer (2)
7.2	$\text{DF}^2 = \text{DE}^2 + \text{EF}^2 - 2\text{DE.EF.cos}\hat{\text{E}}$ $= \left(\frac{2h}{\tan\theta}\right)^2 + \left(\frac{h}{\tan\theta}\right)^2 - 2\left(\frac{2h}{\tan\theta}\right)\left(\frac{h}{\tan\theta}\right)\cos 2\theta$ $= \frac{4h^2}{\tan^2\theta} + \frac{h^2}{\tan^2\theta} - \frac{4h^2}{\tan^2\theta}.\cos 2\theta$ $= \frac{5h^2 - 4h^2\cos 2\theta}{\tan^2\theta}$ $= \frac{h^2\left(5 - 4(2\cos^2\theta - 1)\right)}{\tan^2\theta}$ $= \frac{h^2\left(5 - 8\cos^2\theta + 4\right)}{\tan^2\theta}$ $= \frac{h^2\left(9 - 8\cos^2\theta\right)}{\tan^2\theta}$ $= \sqrt{\frac{h^2\left(9 - 8\cos^2\theta\right)}{\tan^2\theta}}$ $\therefore \text{DF} = \frac{h\sqrt{9 - 8\cos^2\theta}}{\tan\theta}$	✓ subst ✓ simplification ✓ $2\cos^2 - 1$ ✓simplification ✓ $\sqrt{\frac{h^2\left(9 - 8\cos^2\theta\right)}{\tan^2\theta}}$ (5)

7.3	Area $\Delta DEF = \frac{1}{2}.DE.EF.\sin 2\theta$ $= \frac{1}{2}(85{,}78)(42{,}89)\sin 50^\circ$ $= 1409{,}18 m^2$ **OR** Area $\Delta DEF = \frac{1}{2}.2.EF.EF.\sin 2\theta$ $= EF^2 \sin 2\theta$ $= (42{,}89)^2 \sin 50°$ $= 1409{,}18 m^2$	✓ area rule ✓ EF = 42,89 ✓ substitution ✓ answer (4) ✓ area rule ✓ EF = 42,89 ✓ substitution ✓ answer (4)
		[11]

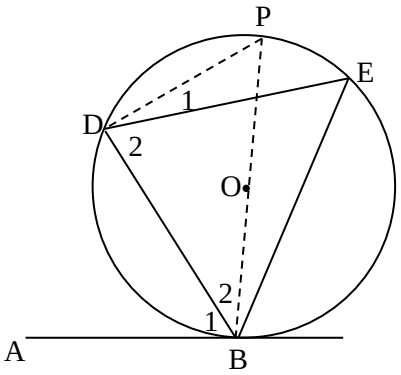
QUESTION 8 **[15]**

8.1



Construction: Draw diameter PB and join P and E. ✓ construction

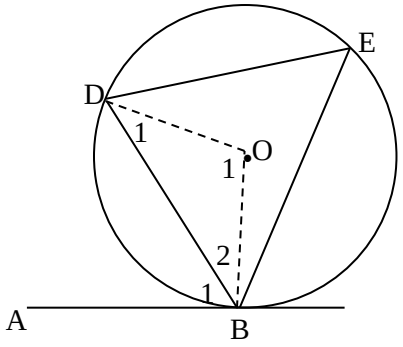
8.1	$\hat{B}_1 + \hat{B}_2 = 90^\circ$	rad $\perp$ tangent	✓S ✓R
	$\hat{E}_1 + \hat{E}_2 = 90^\circ$	$\angle$ in semi-circle	✓S ✓ R
	$\hat{B}_2 = \hat{E}_1$	$\angle$'s in same segment	✓S / R
	$\therefore\ \hat{B}_1 = \hat{E}_2$		(6)



Construction: Draw diameter PB and join P and D. ✓ construction

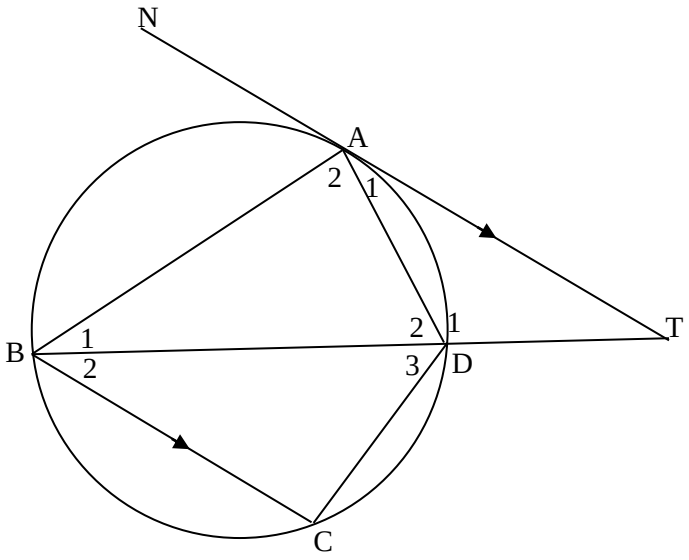
$\hat{B}_1 + \hat{B}_2 = 90^\circ$	rad $\perp$ tangent	✓ S ✓R
$\hat{D}_1 + \hat{D}_2 = 90^\circ$	$\angle$ in semi-circle	✓S / R
$\hat{B}_2 = 90° - \hat{B}_1$		✓ S /R
$\hat{P} = \hat{B}_1$	$\angle$s of ΔBDP	✓R
$\therefore\ \hat{B}_1 = \hat{E}$	$\angle$'s in same segment	(6)

OR



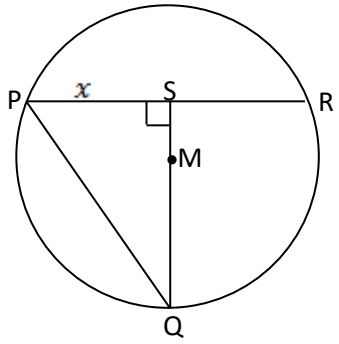
Construction: Join OB and OE		✓ construction
$\hat{B}_1 + \hat{B}_2 = 90^\circ$	rad $\perp$ tangent	✓ S ✓R
$\hat{B}_1 = x$		
$\hat{B}_2 = 90° - x$		
$\hat{B}_2 = \hat{D}_1 = 90° - x$	equal ∠s opp. equal sides	✓S / R
$\hat{O}_1 = 2x$	∠s of ΔBOD	✓ S / R
$\hat{E} = x$	∠ at centre = 2∠ at circum	✓ R
$\therefore\ \hat{B}_1 = \hat{E} = x$		(6)

8.2



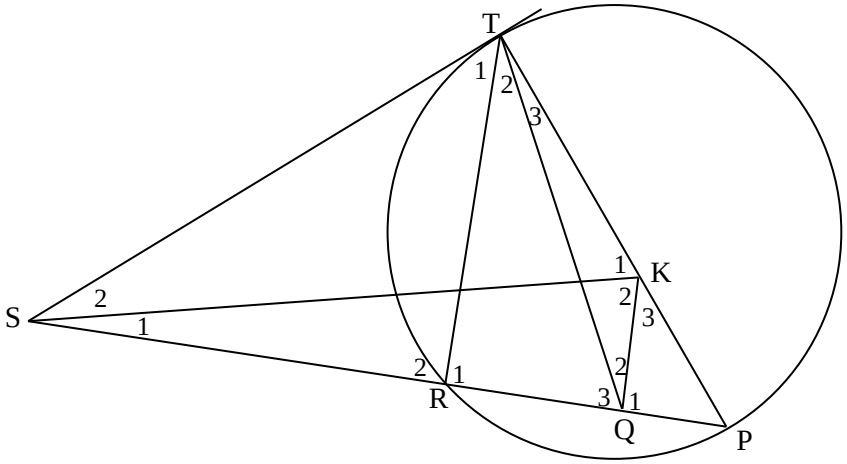
8.2.1	$\hat{A}_1 = 30^\circ$	tangent and chord	✓ S ✓ R (2)
8.2.2	$\hat{A}_2 = 99^\circ$ $\hat{T} = 21^\circ$ **OR** $\hat{T} = 51° - 30°$ $= 21°$	sum of angles of Δ sum of angles of Δ **ext $\angle$ of** ΔAET	✓ S / R ✓ S / R (2) ✓ S / R ✓ S / R (2)
8.2.3	$\hat{T} = \hat{B}_2$ $\hat{B}_2 = 21^\circ$	altern $\angle$'s NT $\parallel$ CD	✓ S / R ✓ answer (2)
8.2.4	$\hat{A}_2 = 99^\circ$ $\hat{C} = 81^\circ$	proven opp. angles of cyclic quad	✓ S ✓ S ✓ R (3)
			[15]

QUESTION 9



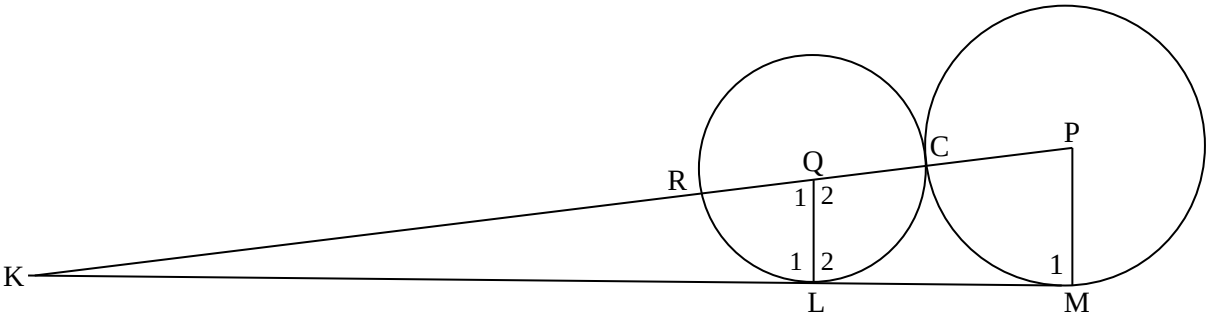
9.1	$PS = SR$ line through centre $\perp$ chord $PQ = PR = 2x$ $PQ^2 = PS^2 + QS^2$ Pythagoras $(2x)^2 = x^2 + QS^2$ $3x^2 = QS^2$ $\sqrt{3}x = QS$	✓ S ✓ R ✓ S / R ✓ $3x^2$ ✓ answer (5)
9.2	$QS = \sqrt{3}\sqrt{12}$ $QS = 6$ $\therefore\ r = 5$	✓ length of QS ✓ answer (2)
9.3	$\sin\hat{M}_1 = \frac{\sqrt{12}}{5}$ $\hat{M}_1 = 43{,}85°$ $\hat{M}_2 = 180° - 43{,}85°$ sum of $\angle$s on a straight line $= 136{,}15°$ $\hat{P} = 68{,}1°$	✓ $\sin\hat{M}_1 = \frac{\sqrt{12}}{5}$ ✓ $\hat{M}_1 = 43{,}85°$ ✓ $\hat{M} = 136{,}15°$ ✓R ✓ answer (5)
		[12]

QUESTION 10



10.1	$\frac{PQ}{PR}=\frac{\sqrt{2}}{\sqrt{18}}=\frac{1}{3}$ $\frac{PK}{PT}=\frac{1x}{3x}=\frac{1}{3}$ RT ‖ QK sides are divided proportionally	✓both ratios ✓R (2)
10.2	$\hat{R}_1 = 90^\circ$ angle in semi-circle $\hat{Q}_1 = 90^\circ$ corresp. angles; RT ‖ QK $\hat{T}_1+\hat{T}_2+\hat{T}_3=90^\circ$ radius ⊥ tangent $\therefore\ \hat{T}_1+\hat{T}_2+\hat{T}_3=\hat{Q}_1$ TKQS is cyclic quad ext ∠ = opp int ∠	✓ S / R ✓ S ✓ R ✓ S / R ✓ S ✓ R (6)
10.3	In ΔKTS and ΔQRT $\hat{K}_1=\hat{Q}_3$ ∠s in the same segment $S\hat{T}K=\hat{R}_1=90°$ proven $\hat{S}_2=\hat{T}_2$ ∠s on a straight line $\therefore \Delta KTS \,|||\, \Delta QRT$ equal angles	✓S / R ✓S ✓S / R ✓R (4)
		[12]

QUESTION 11



11.1	$\frac{QL}{PM}=\frac{2}{3}$ given Let $QL=2r$ and $PM=3r$ $RQ=QC=QL$ radii $RP=RQ+QC+PC$ $=2r+2r+3r$ $=7r$ $\therefore\ RQ:RP=2:7$	✓ S $(RQ=QC=QL)$ ✓ $RP=2r+2r+3r$ (2)
11.2	ΔKMP	✓ S (1)
11.3	$\frac{KL}{KM}=\frac{LQ}{MP}=\frac{KQ}{KP}$ $\Delta KLQ \|\|\| \Delta KMP$ $\frac{2}{3}=\frac{KQ}{KP}$ $2KP=3KQ$ $2(KR+RQ+PQ)=3(KR+RQ)$ $2(KR+2r+5r)=3(KR+2r)$ $2(KR+7r)=3(KR+2r)$ $KR=8r$ $\therefore\ KR:RQ=8r:2r$ $\frac{KR}{RQ}=4$	✓ S ✓ S $\left(\frac{2}{3}=\frac{KQ}{KP}\right)$ ✓ S ✓ answer (4)
		[7]

TOTAL: 150