Education

KwaZulu-Natal Department of Education

REPUBLIC OF SOUTH AFRICA

MATHEMATICS P2

MARKING GUIDELINE

PREPERATORY EXAMINATION 2017

NATIONAL SENIOR CERTIFICATE

GRADE 12

MARKS: 150

N.B. This marking guideline consists of 14 pages.

QUESTION 1

1.1

	f	Midpoint(x)	$f.x$
$120<x\leq 135$	1	127.5	127.5
$135<x\leq 150$	15	142.5	2137.5
$150<x\leq 165$	45	157.5	7087.5
$165<x\leq 180$	28	172.5	4830
$180<x\leq 195$	1	187.5	187.5
TOTAL	90		14370

159,6

OR

$$Mean\ \ height = \frac{14370}{90}$$
$$= 159{,}67$$

Marking:

Answer only – full marks

✓A table method

✓CA answer (2)

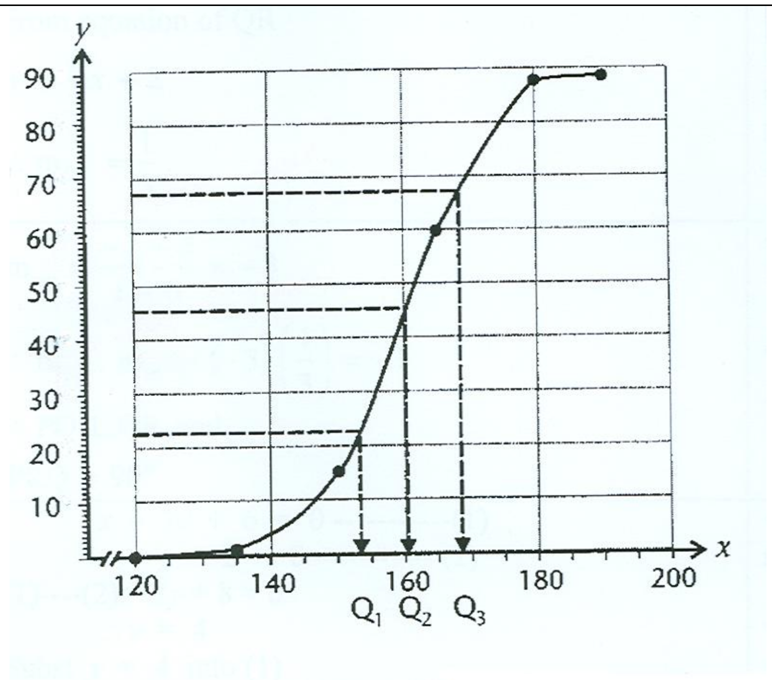
OR

✓14370

✓ answer

(2)

1.2



Marking:

✓ A shape

✓ A (120;0)

✓ A (60;165)

✓ A for any other correct plotted points

(4)

1.3

1.3.1 median height = 161 cm

✓✓ A A Answer

(Accept : 160 – 163)

(2)

1.3.2

IQR $= Q_3 - Q_1$

$= 169 - 152$

$= 17$ cm

✓ CA Q_3

✓ CA Q_1

✓CA answer

(Accept 1 mark deviation for Q_1 and Q_3)

(3)

[11]

QUESTION 2

2.1	(40; 13)	✓ A (40; 13) (1)
2.2.1	$y = 0,04x + 8,64$	✓CA 0,04x ✓ CA 8,64 ✓ CA equation (3)
2.2.2	$r = 0,91$ strong, positive correlation	✓ CA 0,91 ✓ CA justification (2)
2.2.3	The car will be using about 11,69 *l*/100 km.	A✓CA✓ average 11,69 *l*/100 km (2)
		[8]

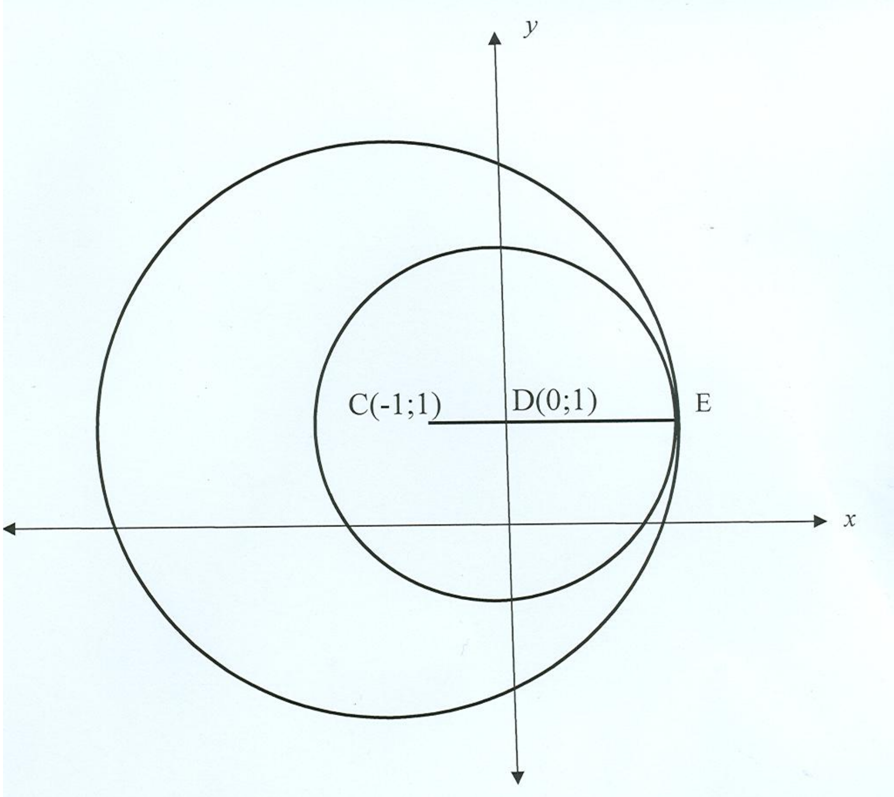
QUESTION 3

3.1	$0-3y+6=0$ $-3y=-6$ $y=2$ $\therefore$ Q(0;2)	✓A subst $x = 0$ ✓A $y = 2$ (2)
3.2	$y=\frac{1}{3}x+2$ $\therefore m_{QR}=\frac{1}{3}$	✓ A writing in standard form ✓ A answer (2)
3.3	$m_{PQ}=\frac{-1-2}{1-0}=-3$ $\therefore m_{PQ}\times m_{QR}=(-3)\left(\frac{1}{3}\right)=-1$ $\therefore PQ\perp QR$ Thus $P\hat{Q}R=90^\circ$	✓ A gradient of PQ ✓ A products of gradients (2)
3.4	$x-3y+6=0$........(1) $x-y-2=0$...........(2) $(1)-(2):-2y+8=0$ $y=4$ subst $y=4$ into (1) $x-3(4)+6=0$ $x-12+6=0$ $x=6$ R(6;4)	✓ M solving both equations simultaneously ✓ CA substituting $y = 4$ ✓ CA $x = 6$ (provided R is in first quadrant) (3)

3.5	$QR=\sqrt{(0-6)^2+(2-4)^2}=2\sqrt{10}$ $PQ=\sqrt{(0-1)^2+(2+1)^2}=\sqrt{10}$ Area of $\Delta PQR=\frac{1}{2}QR\times PQ$ $=\frac{1}{2}\left(2\sqrt{10}\right)\left(\sqrt{10}\right)$ $=10$ square units	✓ CA QR value ✓ A PQ value ✓ CA correct substitution into area formula ✓CA answer (4)
3.6	$PR^2=(6-1)^2+(4-(-1))^2$ $=5^2+5^2=50$ $\therefore PR=\sqrt{50}=5\sqrt{2}$ units	✓CA correct subst. into distance formula ✓ CA answer (2)
		[15]

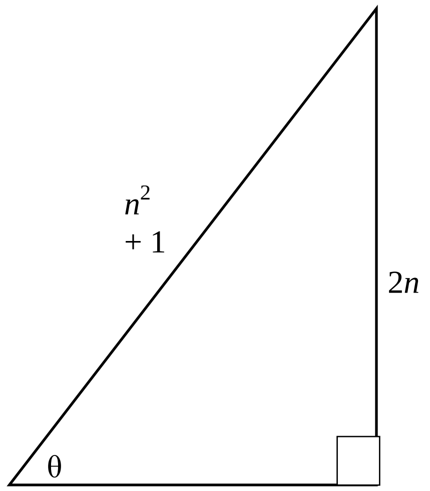
QUESTION 4

4.1.1	$m_{MN}\times m_{NP}=-1$ Radius ⊥ Tangent $\left(\frac{a-1}{2+2}\right)\times\left(\frac{a+7}{2+2}\right)=-1$ $\left(\frac{a-1}{4}\right)\left(\frac{a+7}{4}\right)=-1$ $\frac{a^2-a+7a-7}{16}=-1$ $a^2+6a-7=-16$ $a^2-6a+9=0$ $(a+3)(a+3)=0$ $\therefore a=-3$	✓ A $m_{MN}\times m_{NP}=-1$ ✓ A substitution ✓A simplification ✓A standard form ✓A factorization (5)
4.1.2	$MN^2=r^2=(-2-2)^2+(1+3)^2=32$ $(x+2)^2+(y-1)^2=32$	✓ A value of radius ✓CA equation of circle (2)
4.1.3	M(– 2; 1) N(2 ; – 3) Q(– 6; – 3) $m_{MN}=\frac{1+3}{-2-2}=-\frac{4}{4}=-1$ $\therefore m_{PN}=1$ [tangent at N] $m_{MQ}=\frac{1+3}{-2+6}=\frac{4}{4}=1$ $\therefore m_{PQ}=-1$ [tangent at Q]	✓A gradient of MN ✓CA gradient of PN ✓ A gradient of MQ ✓CA gradient of PQ (4)

4.1.4	$\tan\theta = 1$ $\tan\alpha = -1$ $\tan^2\alpha + \tan^2\theta$ $= (1)^2 + (-1)^2$ $= 2$	✓A $\tan\theta = 1$ ✓A $\tan\alpha = -1$ ✓A Substitution (1) ✓A Substitution (-1) (4)
4.2.1	 Circle centre C: C(−1 ; 1) and radius = 4 units Circle centre D: $x^2 + (y-1)^2 = 9$ D(0;1) and radius = 3 units Difference of radii = 4 − 3 = 1 unit $CD^2 = (-1-0)^2 + (1-1)^2 = 1$ $\therefore CD = 1$ unit (CD < $r_1 + r_2$ therefor the circles touch internally) Therefore the two circles touch each other internally	✓A centre and radius of circle centre C ✓ A equation of circle center D ✓ CA centre and radius of circle centre D ✓A Difference of radii = 1 unit ✓length of CD (5)

4.2.2	$x^2+2x+1+y^2-2y+1=16$ $x^2+2x+y^2-2y=14$..........(1) $x^2+y^2 \quad -2y=8$ ....(2) $(1)-(2)$ $2x=6$ $x=3$ $\therefore$ The equation of the common tangent is: $x=3$	✓M solving simultaneously ✓A answer (2)
		[22]

QUESTION 5

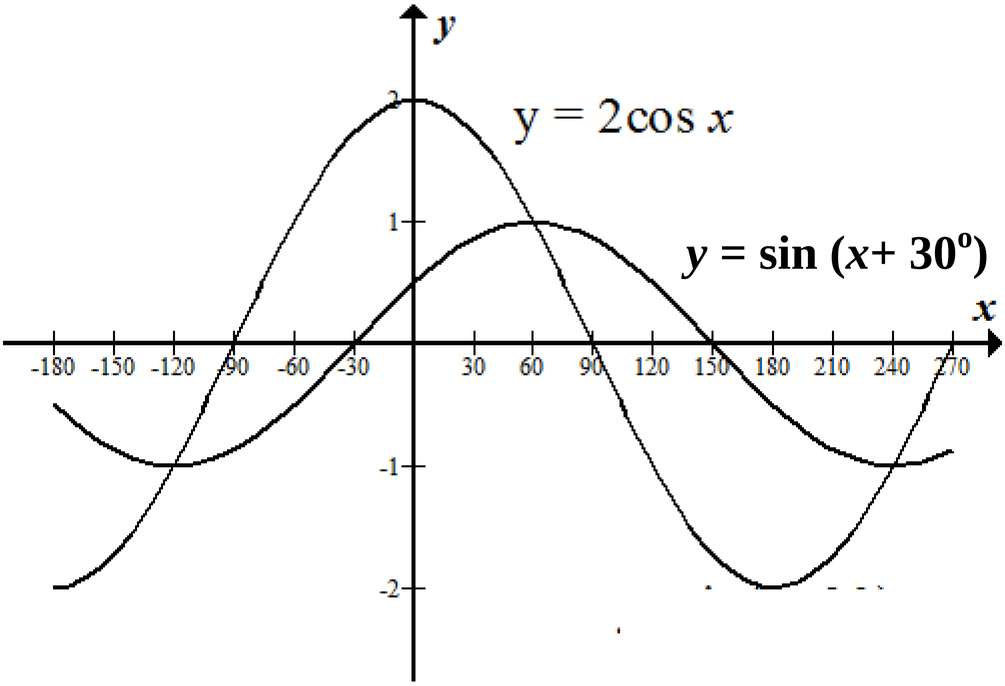
5.1	$\sqrt{2}\cos(-45^o)+\cos 210^o-\tan 840^o$ $=\sqrt{2}\cos 45^\circ+(-\cos 30^\circ)-(-\tan 60^\circ)$ $=\sqrt{2}\,.\,\frac{\sqrt{2}}{2}-\cos 30^o+\tan 60^o$ $=1-\frac{\sqrt{3}}{2}+\frac{\sqrt{3}}{1}$ $=\frac{2-\sqrt{3}+2\sqrt{3}}{2}$ $=\frac{2+\sqrt{3}}{2}$	✓A $\cos 45^\circ$ ✓ A $-\cos 30^\circ$ ✓ A $-\tan 60^\circ$ ✓A substitution of special angle values ✓ A simplification (5)
5.2	 $\sqrt{(n^2+1)-(2n)^2}$ $=\sqrt{n^4-2n^2+1}$ $=\sqrt{(n^2-1)^2}$ $=n^2-1$ $\frac{1+\sin\theta}{\cos\theta}=\frac{1+\frac{2n}{n^2+1}}{\frac{n^2-1}{n^2+1}}$ $=\frac{n^2+1+2n}{n^2+1}\times\frac{n^2+1}{n^2-1}$ $=\frac{(n+1)^2}{n^2-1}$ $=\frac{(n+1)(n+1)}{(n-1)(n+1)}=\frac{n+1}{n-1}$	✓ A Drawing the sketch ✓ A calculating remaining side ✓ A substitution sinθ value ✓ A substitution cosθ value ✓numerator simplification ✓factorization : n^2+1+2n ✓factorization : n^2-1 (7)

5.3	$\dfrac{\sin 2x}{\cos x(1-\cos 2x)(1+\frac{1}{\tan^2 x})} = \sin x$ LHS $= \dfrac{\sin 2x}{\cos x(1-\cos 2x)(1+\frac{1}{\tan^2 x})}$ $= \dfrac{\frac{2\sin x\cos x}{\cos x}}{\left(1-(1-2\sin^2 x)\right)(1+\frac{\cos^2 x}{\sin^2 x})}$ $= \dfrac{2\sin x}{(2\sin^2 x)(\frac{\sin^2 x+\cos^2 x}{\sin^2 x})}$ $= \dfrac{2\sin x}{2\sin^2 x\ .\ \frac{1}{\sin^2 x}}$ $= \sin x$ $=$ RHS	✓ A $\sin 2x = 2\sin x\cos x$ ✓ A $\tan x = \dfrac{\sin x}{\cos x}$ ✓ A $\cos 2x = 1-2\sin^2 x$ ✓ A simplification - numerator ✓ A simplification - denominator (5)
5.4	Area Triangle AOB= $\frac{1}{2}$ ab sin $\hat{AOB}$ $= \frac{1}{2} x\ .\ x \sin 90^o$ (Area maximum if $\hat{AOB} = 90^o$) $= \frac{1}{2}x^2\ .1$ $= \frac{1}{2}x^2$	✓ A correct substitution into formula ✓ A Max area if $\hat{AOB} = 90^o$ ✓A Answer (3)
		[20]

QUESTION 6

6.1.1	$\sin(x+30^\circ)$ $= \sin x\cos 30^\circ + \cos x\sin 30^\circ$ $= \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$	✓ A expanding ✓ A $\cos 30^\circ = \frac{\sqrt{3}}{2}$ ✓ A $\sin 30^\circ = \frac{1}{2}$ (3)
6.1.2	$2\cos x = \frac{\sqrt{3}}{2}\sin x + \frac{1}{2}\cos x$ $\frac{3}{2}\cos x = \frac{\sqrt{3}}{2}\sin x$ $\frac{3}{2}\cos x = \frac{\sqrt{3}}{2}\sin x$ dividing both sides by cos x; cos $\neq 0$ $\frac{\sin x}{\cos x} = \frac{3}{2}$ x $\frac{2}{\sqrt{3}}$ $\tan x = \frac{3}{\sqrt{3}}$ $= \frac{3\sqrt{3}}{3}$ $= \sqrt{3}$ $\therefore \quad x = 60^\circ + k.180^\circ, k \in \mathbb{Z}$ $\therefore \quad x \in \{-120^\circ; 60^\circ; 240^\circ\}$	✓ CA $\frac{3}{2}\cos x$ ✓ A $\tan x$ ✓ CA $\sqrt{3}$ ✓ CA $x = 60^\circ + k.180^\circ, k \in \mathbb{Z}$ ✓ CA 120° ✓ CA 60° ✓ CA 240° (7)

6.2



✓A✓A x - intercepts of EACH graph
✓A✓A - maximum and minimum values of EACH graph
✓A✓A - shape of EACH graph

(6)

6.3	$\frac{KT}{KM} = \tan\beta$ $\therefore$ KT = KM $\tan\beta$ ....(1) $\frac{KM}{\sin y} = \frac{KL}{\sin(180^\circ - (x+y))}$ $\therefore$ $KM = \frac{KL\sin y}{\sin(x+y)}$ Substituting KM in equation (1) give $KT = \frac{KL\sin y.\tan\beta}{\sin(x+y)}$	✓writing $\tan\beta = \frac{KT}{KM}$ (A) ✓A making KT the subject of the formula. ✓M applying sine rule ✓A sin (*x* + y) ✓A substituting in equation 1 (5)
		[21]

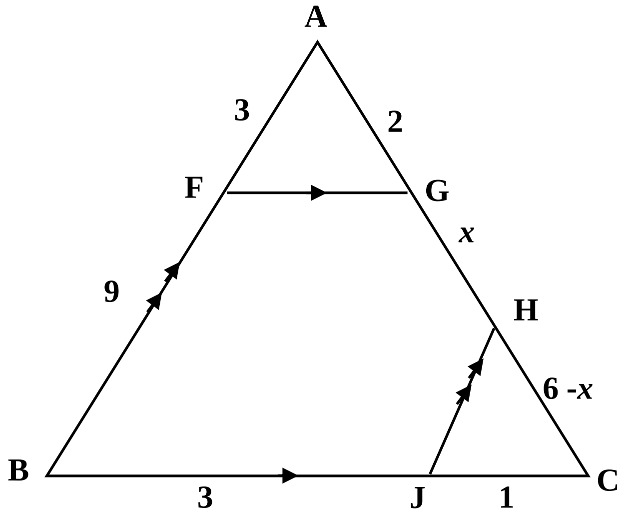
QUESTION 7

7	1. $\hat{B}_2 = x = \hat{A}_2$ ($\angle$s opp = sides) 2. $\hat{A}_1 = x = \hat{B}_2$ ....(tan chord theorem) 3. $\hat{A}_1 = \hat{S}_1 = x$ ....(corres $\angle$s, DA || CS) 4. $\hat{B}_3 = x$ (alternate angles, DA || CS) 5. $C\hat{D}A = x$ ……..(Exterior angle of a cyclic quad)	S✓/R✓ S✓/R✓ S/R✓ S✓/R✓ S✓ R✓ **(All Accuracy)** (Penalize once for not stating parallel lines) (9)
		[9]

QUESTION 8

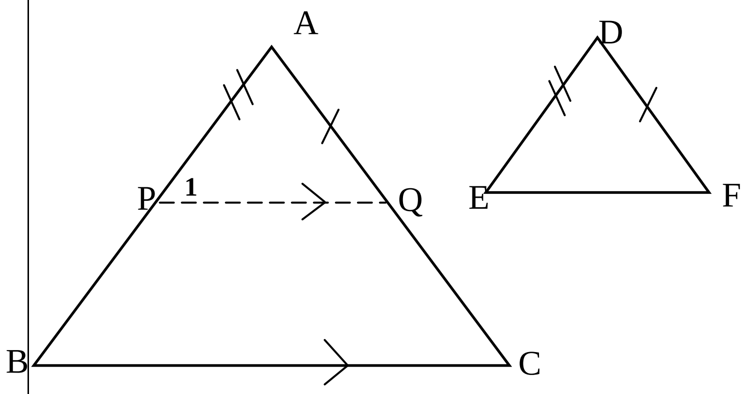
8.1	$\hat{Z}_2 = 90^o$ $\hat{Z}_2 = \hat{O}_1$ $\therefore$ VOYZ is a cyclic quadrilateral ... (Converse opp angles of a cyclic quad)	✓S ✓R angle in semi circle each $= 90^o$ ✓Reason (3)
8.2	$\hat{Z}_1 = \hat{Y}$ (tan. chord theorem) $\hat{V}_1 = \hat{Y}$ (ext cyclic quad) $\hat{Z}_1 = \hat{V}_1$ $\therefore \Delta$ WVZ is isosceles (two equal angles)	✓S/R ✓ S/R ✓R (3)
8.3	In Δ XOV and ΔXZY $\hat{X}$ is common $\hat{O}_1 = \hat{Z}_2 = 90^\circ$ (ext cyclic quad) $\therefore \hat{V}_2 = \hat{Y}$ (remaining angles) $\therefore \Delta$ XOV $\vert\vert\vert$ ΔXZY $\angle\angle\angle$	✓ S ✓S /R ✓ S/R ✓ R ($\angle\angle\angle$) (4)
8.4	$\therefore \frac{XO}{XZ} = \frac{VO}{ZY}$ ($\vert\vert\vert$ Δs) $\frac{10}{16} = \frac{VO}{12}$ $VO = \frac{10 \times 12}{16}$ $\therefore$ VO = 7,5 units	✓S $\frac{XO}{XZ} = \frac{VO}{ZY}$ ✓substitution ✓answer (3)
		[13]

QUESTION 9



9.1	$\frac{FA}{FB} = \frac{AG}{GC}$ $\left(Prop\ intercept\ theorem, FG \parallel BC\right)$ $\frac{3}{9} = \frac{2}{GC}$ $3GC = 18$ $\therefore GC = 6$	✓S/✓R ✓Substitution ✓ Answer (4)
9.2	Let GH $= x$ $\therefore$ HC $= 6 - x$ $\frac{6-x}{x+2} = \frac{1}{3}$... (HJ // AB prop theorem) $18 - 3x = x + 2$ Now $-4x = -16$ $\therefore x = 4$ $\therefore$ GH = 4 units	(CA applies in the question) ✓M S✓R✓ ✓Simplification ✓ Answer (5)
		[9]

QUESTION 10

<table>
<tr><td>10.1</td><td>Given: $\hat{A} = \hat{D}$; $\hat{B} = \hat{E}$ and $\hat{C} = \hat{F}$
RTP: $\frac{AB}{DE} = \frac{AC}{DC} = \frac{BC}{EF}$
Construction: mark P on Q so that AP = DE and AQ = DF


Proof: Δ APQ ≡ Δ DEF … (S; ∠, S)
$\therefore \hat{P}_1 = \hat{E} = P\hat{B}C$
PQ//BC …. Corresponding ∠'s $\hat{P}_1 = P\hat{B}C$
$\therefore \frac{AB}{AP} = \frac{AC}{AQ}$ (prop. theorem PQ//BC)
$\therefore \frac{AB}{DE} = \frac{AC}{DF}$
Similarly, Mark P and R on AB and AC
Such that BP = ED and BR = EF
$\frac{BA}{BP} = \frac{BC}{BR}$ and $\frac{BA}{ED} = \frac{BC}{EF}$
$\therefore \frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF}$</td><td>✓ Construction
✓ S/✓R
✓ S
✓ S/R
✓ S/R
✓ S
(7)</td></tr>
<tr><td>10.2.1</td><td>In ΔTDA and ΔFDB
$\hat{A}_2 = \hat{B}_3$ (angles in same segment)
$\hat{F} = A\hat{T}D$ (angles in same segment)
$A\hat{D}T = F\hat{D}B$ (Vertically Opposite angles)
∴ ΔTDA ||| ΔFDB (∠∠∠)</td><td>✓S/✓R
✓S/R
✓3 ∠'s (4)</td></tr>
</table>

10.2.2	$\Delta TDA \parallel\!\mid \Delta FDB$ $\therefore \frac{AD}{BD} = \frac{TD}{FD}$ (||| Δs) $\therefore AD.FD = BD.TD$ $= BD^2$ $\therefore TB^2 = (2BD)^2$ $TB^2 = 4BD^2$ $TB^2 = 4AD.FD$	✓S ✓ S (2)
10.2.3	ΔBDE and ΔADB 1. $\hat{D}_2$ is common 2. $\hat{B}_2 = A_1$ (tan chord theorem) 3. $\hat{E}_1 = A\hat{B}D$ (3rd $\angle$ Δ) $\therefore \Delta BDE \parallel\!\mid \Delta ADB$ ($\angle\angle\angle$) $\frac{BD}{AD} = \frac{DE}{BD}$ (||| Δs) $BD^2 = DE.\ AD$	✓ S ✓ S/R ✓ S/R ✓ S (4)
10.2.4	$\left(\frac{1}{2}TB\right)^2 = DE.\ AD$ $TB^2 = 4DE.AD$ $4AD\ .\ FD = 4AD\ .\ DE$ $\therefore FD = DE$ In Δ DET and ΔDFB 1. FD = DE 2. $E\hat{D}T = B\hat{D}F$ (vert opp $\angle$s) 3. BD = DT $\therefore \Delta DET \equiv \Delta DFB$ (SAS) $\therefore ET = FB$ ($\equiv \Delta s$)	✓S ✓S ✓S ✓all three statements ✓S/R (5)
		[22]

TOTAL MARKS: 150