**NATIONAL
SENIOR CERTIFICATE**

GRADE 12

MATHEMATICS P1

MEMORANDUM

SEPTEMBER 2018

MARKS: 150

TIME: 3 hours

This memorandum consists of 16 pages.

NOTE:
- If a candidate answered a question TWICE, mark only the FIRST attempt.
- If a candidate crossed out an answer and did not redo it, mark the crossed-out answer.
- Consistent accuracy applies to ALL aspects of the marking memorandum.
- Assuming values/answers in order to solve a problem is unacceptable.

QUESTION 1

1.1.1	$(x-3)(x+1) = 0$ $x - 3 = 0$ or $x + 1 = 0$ $x = 3$ or $x = -1$	Values/*waardes of/van* x ✓ $x = 3$ ✓ $x = -1$ (2)

1.1.2	$x^2 - x - 4 = 0$
	
	 $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
	
	 $= \dfrac{1 \pm \sqrt{(-1)^2 - 4(1)(-4)}}{2(1)}$
	
	 $x = \dfrac{1 \pm \sqrt{17}}{2}$
	 $x = 2{,}56$
	
	 or $x = -1{,}56$
	
	 ALTERNATIVE /ALTERNATIEF
	 $x^2 - x - 4 = 0$
	
	 $(x - \dfrac{1}{2})^2 = 4\dfrac{1}{4}$
	
	 $x = \dfrac{1}{2} \pm \dfrac{\sqrt{17}}{2}$
	 $x = 2{,}56$ or/of $x = -1{,}56$
		✓ standard form
		   /*standaardvorm*
		
		 ✓ correct substitution into correct formula
		   /*korrekte vervanging in korrekte formule*
		
		 ✓ $x = 2{,}56$
		 ✓ $x = -1{,}56$ (5)
		
		 OR
		
		 ✓ standard form/*std vorm*
		 ✓ completing the square
		   /*voltooiing van vierkant*
		
		 ✓ $x = 2{,}56$
		 ✓ $x = -1{,}56$ (4)

| 1.1.3 | $x + \sqrt{x-2} = 4$ $$\left(\sqrt{x-2}\right)^2 = (4-x)^2$$ $$x - 2 \ = \ 16 - 8x + x^2$$ $$x^2 - 9x + 18 = 0$$ $$\left(x-3\right)\left(x-6\right)=0$$ $$x = 3 \ \text{or/of} \ x = 6$$ $$\therefore x = 3$$

 ALTERNATIVE/ALTERNATIEF

 $$\left(\sqrt{x-2}\right)^2 = (4-x)^2$$ $$x - 2 = 16 - 8x + x^2$$ $$x^2 - 9x + 18 = 0$$ $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ $$= \frac{1 \pm \sqrt{(-9)^2 - 4(1)(18)}}{2(1)}$$ $$x = \frac{1 \pm \sqrt{9}}{2}$$ $$x = 3 \ \text{ or } \ x = 6$$ $$\therefore x = 3$$ | ✓ squaring both sides /*kwadrering beide kante*

 ✓ standard form/*std vorm*
 ✓ factors/*faktore*

 ✓ both values/beide waardes
 ✓ select/*selekteer* $x = 3$ (5)
 OR

 ✓squaring both sides /*kwadrering beide kante*

 ✓ standard form/*std vorm*

 ✓formula/*formule*

 ✓both values/*beide waardes*

 ✓ select/*selekteer* $x = 3$

 (5) |
| 1.1.4 | $x \le 0 \ \text{ or } \ x \ge \dfrac{1}{2}$ | ✓critical values /*kritieke waardes*
 ✓✓both answers and inequality signs correct (3) /*beide antwoorde en ongelykheids tekens moet reg wees* |

1.2.1		
	$P = \dfrac{4^{x+3} + 4^x}{8^{x+2} + 8^x}$	✓prime numbers with exponents/*priemgetalle met eksponente*
	$= \dfrac{2^{2x+6} + 2^{2x}}{2^{3x+6} + 2^{3x}}$	
	$= \dfrac{2^{2x}(2^6 + 1)}{2^{3x}(2^6 + 1)}$	✓ factorisation/*faktorisering* ✓ simplification /*vereenvoudiging*
	$= 2^{-x}$ or/of $\left(\dfrac{1}{2}\right)^x$	✓ answer/*antwoord* (4)
	ALTERNATIVE/ALTERNATIEF	**OR**
	$P = \dfrac{4^x\left(4^3 + 1\right)}{8^x\left(8^2 + 1\right)}$	✓ factorisation/*faktorisering* ✓ factorisation/*faktorisering*
	$= \dfrac{4^x(65)}{8^x(65)}$	✓simplification/*vereenvoudiging*
	$= \left(\dfrac{1}{2}\right)^x$	✓answer/*antwoord* (4)

1.2.2	$P = 8$	✓ equating/*stel gelyk*
	$2^{-x} = 8$	
	$2^{-x} = 2^3$	
	$\therefore -x = 3$	✓answer/*antwoord*
	$\therefore x = -3$	(2)

1.3	$3y = -3 + x + x^2$	
	$\sum_{k=3}^{4} x(k-2)^2 = 5y$	
	$k = 3: \ x(3-2)^2 = x$	✓
	$k = 4: \ x(4\text{-}2) = 4x$	both values in terms of x /beide waardes in terme van
	$\therefore x + 4x = 5y$	
	$5x = 5y$	
	$\therefore x = y$	✓ equating/stel gelyk
	$(1)...3x = -3 + x + x^2$	
	$0 = x^2 - 2x - 3$	
	$0 = (x+1)(x-3)$	
	$\therefore x = -1 \quad \text{or} \quad x = 3$	✓ standard form/standaardvorm
	$y = -1 \ \text{or} \ y = 3$	
		✓ $x = -1$ or $x = 3$
		✓ $y = -1$ or $y = 3$
		(5)

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QUESTION 2/VRAAG 2

2.1	$10 - 3y \ ; \ 7 \ ; \ 15 \ ; \ 8y + 1.$	
	$10 - 3y \ ; \quad 7 \quad ; \quad 15 \quad ; \ 8y + 1$	
	$-3 + 3y \qquad 8 \qquad 8y - 14$	✓ First differences /eerste verskil
		✓ Second differences /tweede verskil
	$11 - 3y \quad 8y - 22$	✓ equating /gelykstelling
	$11 - 3y = 8y - 22$	✓ value of y /waarde van
	$\therefore y = 3$	(4)

2.2	$1 \; ; \; 7 \; ; \; 15; \; 25$	✓The quadratic row /kwadratiese ry
	$2a = 2$	
	$\therefore a = 1$	✓ value of a /waarde van
	$6 = 3(1) + b$	
	$3 = b$	✓value of b /waarde van
	$1 + 3 + c = 1$	
	$\quad c = -3$	✓value of c /waarde van
	$T_n = n^2 + 3n - 3$	(4)
2.3	$n^2 + 3n - 3 > 2157$	✓inequality /ongelykheid
	$n^2 + 3n - 2160 > 0$	✓ factorization faktorisering
	$(n+48)(n-45) > 0$	
	$n < -48 \;$ or $\; n > 45$	✓ select correct value of n /selekteer korrekte waarde
	$\therefore T_{46}$	(3)
	ALTERNATIVE:	
	$n^2 + 3n - 3 > 2157$	**OR**
		✓ inequality /ongelykheid
	$n^2 + 3n - 2160 > 0$	
	$n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	
	$n = \dfrac{-3 \pm \sqrt{8649}}{2}$	✓ formula /formule
	$n = -48 \;$ or/of $\; n = 45$	
		✓ select correct value of n /selekteer regte waarde (3)
	$\therefore T_{46}$	

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QUESTION 3/VRAAG 3

3.1.1	The first term $a = p$ and the constant difference $d = q - p$	✓ p and $(q-p)$
		✓ $9q - 8p$
	$T_{10} = p + 9(q-p) = 9q - 8p$	(2)

3.1.2	$S_n = \dfrac{n}{2}\big[2a + (n-1)d\big]$ $\therefore S_{20} = 10(2a - 19d)$ $S_{20} = 20p + 190(q - p)$ $\qquad = 190q - 170p$	✓correct substitution into correct formula /*korrekte vervanging in korrekte waarde* ✓answer /*antwoord* (2)
3.2.1	$T_3 + T_4 = 167$ $a + 2d + a + 3d = 167$ $2a + 5d = 167$ ....(1) $T_{21} = -4$ $a + 20d = -4$ $a = -20d - 4$ ....(2) Substitute (1) into (2): $2(-20d - 4) + 5d = 167$ $-40d - 8 + 5d = 167$ $-35d = 175$ $d = -5$	✓equation/*vergelyking* 1 ✓ equation/*vergelyking* 2 ✓equating : Equation 1 = equation 2 ✓ value of d (4) /*waarde van*
3.2.2	$a + 20d = -4$ $a = -20(-5) - 4 = 96$ $S_{21} = \dfrac{21}{2}\big[2a + 20d\big]$ $\qquad = \dfrac{21}{2}\big[2(96) + 20(-5)\big]$ $\qquad = 966$ OR $a + 20d = -4$ $a = -20(-5) - 4 = 96$ $S_{21} = \dfrac{21}{2}(a + l)$ $\qquad = \dfrac{21}{2}(96 - 4)$ $\qquad = 966$	✓ value of a /*waarde van* ✓correct substitution in correct formula /*korrekte vervanging in korrekte waarde* ✓ answer/*antwoord* OR ✓ value of a /*waarde van* ✓correct substitution in correct formula /*korrekte vervanging in korrekte waarde* ✓ answer/*antwoord* (3)

| 3.3 | $a = -3; \ r = -\dfrac{1}{3}$

 Sequence is: $-3 \ ; 1 \ ; \ -\dfrac{1}{3} \ ; \ \dfrac{1}{9} \ ; \ -\dfrac{1}{27}$

 Sequence of negative numbers:

 $-3 \ ; \ -\dfrac{1}{3} \ ; \ -\dfrac{1}{27} \ ;......$

 $a = -3 \ ; \ r = \dfrac{1}{9}$

 $\qquad S_\infty = \dfrac{a}{1-r}$

 $\qquad\qquad \dfrac{-3}{1-\dfrac{1}{9}}$

 $\qquad\qquad = \dfrac{-3}{1}\left(\dfrac{9}{8}\right)$

 $\qquad\qquad = \dfrac{-27}{8}$

 $\qquad\qquad = -3\dfrac{3}{8}$ | ✓geometric sequence
 /*meetkundige ry*

 ✓sequence of negative numbers
 /*ry met negatiewe getalle*

 ✓ value of r and a
 /*waarde van*

 ✓correct substitution in correct formula
 /*korrekte vervanging in korrekte formule*

 ✓correct answer
 /*korrekte antwoord* (5) |

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QUESTION 4/VRAAG4

4.1	$y = 2^x$ $\qquad\qquad x = 0$ $\qquad\qquad \therefore f(0) = 2^0 = 1$ $\qquad\qquad \therefore C(0;1)$ y - intercept: $\begin{aligned} g(x) &= -(x-1)^2 + q \\ &C(0;1) \\ 1 &= -(0-1)^2 + q \\ 1 &= -1 + q \\ \therefore q &= 2 \end{aligned}$	✓ substitute $C(0;1)$ /*vervang* ✓ $q = 2$ (2)
4.2	$g(x) = -(x-1)^2 + 2$ D(1; 2)	✓✓answer /*antwoord* (2)
4.3	$t = 2$	✓ answer/*antwoord* (1)

4.4	$f^{-1}: x = 2^y$ $\therefore y = \log_2 x$	✓✓ answer/*antwoord* (2)
4.5		✓ form of the graph /*vorm van die grafiek* ✓ $x-$intercept/*afsnit* ✓ any other coordinate on the graph /*enige ander koordinaat op die grafiek* (3)
4.6	$g(x) = -(x-1)^2 + 2$ $g(x+1) - 2 = -(x-1+1)^2 + 2 - 2$ $\therefore h(x) = -x^2$	✓ $+1$ and -2 ✓ answer/*antwoord* (2)
4.7	Domain: $x \geq 0$ or $x \leq 0$	✓ answer /*antwoord* (1)

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QUESTION 5/VRAAG 5

5.1	$x = -2$ $y = 1$	✓ answer/*antwoord* ✓ answer/*antwoord* (2)
5.2 .1	A : $x = 0$ $y = \dfrac{4}{0+2} + 1$ $y = 2 + 1$ $y = 3$ A(0 ; 3)	✓ answer /*antwoord* (1)

5.2.2	$y = 0$ $0 = \dfrac{4}{x+2} + 1$ $-1(x+2) = 4$ $-x - 2 = 4$ $x = -6$ $D(-6\,;\,0)$	✓ equating to 0 /*gelykstel aan* ✓ simplification /*vereenvoudiging* ✓ answer/*antwoord* (3)
5.2.3	$\dfrac{4}{x+2} + 1 = 2x + 3$ $4 + 1(x+2) = (2x+3)(x+2)$ $4 = 2x^2 + 6x + 4$ B : $0 = 2x^2 + 6x$ $0 = 2x(x+3)$ $\therefore x = 0 \quad \text{or} \quad x = -3$ $g(-3) = 2(-3) + 3 = -3$ $\therefore B(-3;-3)$	✓ equating /*gelykstelling* ✓ standard form /*std vorm* ✓ value of x /*waarde van* ✓ value of y /*waarde van* (4)
5.3	Average gradient: $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ $= \dfrac{0-3}{-6-0} = \dfrac{1}{2}$	✓ applying $\dfrac{y_2 - y_1}{x_2 - x_1}$ /*toepassing* ✓ answer /*antwoord* (2)
5.4	$g(x) = 2x + 3$ $g'(x) = 2$ $\therefore f(x).g'(x) \le 0$ $g'(x) > 0, \forall\ x \in R$ $f(x) \le 0 \quad for$ $-6 \le x < -2$	✓ $g'(x)$ ✓ explaining $g'(x) \le 0$ /*verduidelik* ✓ answer of $f(x) \le 0$ /*antwoord* (3)

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QUESTION 6/VRAAG 6

6.1	$12\,500\left(1-i\right)^5 = 5\,546,32$ $\left(1-i\right)^5 = 0,4437056$ $\sqrt[5]{\left(1-i\right)^5} = \sqrt[5]{0,4437056}$ $1-i = 0,8500001102$ $i = 0,1499998898$ $i \approx 15\%$	✓correct substitution in correct formula */korrekte vervanging in korrekte formule* ✓applying $\sqrt[5]{}$ */toepassing* ✓ value of i (3) */waarde van*
6.2.1	$i_{eff} = (1+\dfrac{i_m}{m})^m$ $1+i_{eff} = (1+\dfrac{0,08}{12})^{12}$ $i_{eff} = 0,082999 \times 100$ $i_{eff} = 8,299 \approx 8,3$	✓ $m=12$ ✓correct substitution in correct formula */korrekte vervanging in korrekte formule* ✓simplification/*vereenvoudig* ✓answer/*antwoord* (4)
6.2.2	Deposit paid $= 1\,800\,000\ \dfrac{60}{100} = R1\,080\,000$ *Deposito betaal* Balance owed = OR $180\,000 \times \dfrac{40}{100} = R720\,000$ Therefore/*dus* : R720 000 was financed by the bank was paid by the bank. *Was gefinansier deur die bank /betaal deur die bank.* $720\,000\left(1+\dfrac{0,08}{12}\right) = x\left[\dfrac{1-\left(1+\dfrac{8}{1\,200}\right)^{(-119)}}{\dfrac{8}{1\,200}}\right]$ $4832 = (0,5464730496\ )x$ $8842,16 = x$	✓ deposit paid/*deposito betaal* ✓ balance owed */balans verskuldig* ✓ correct substitution in correct formula */korrekte vervanging in korrekte formule* ✓ $n=-119$ ✓ answer of R8 842,16 (5) */antwoord*

6.2.3	$A = 724800\left(1 + \dfrac{8}{1\,200}\right)^{(3\times12)}$	✓ correct substitution in correct formula /korrekte vervanging in korrekte formule
	A = R920 667,815	
	$F = 8\,842,16\left[\dfrac{\left(1+\dfrac{8}{1\,200}\right)^{36}-1}{\dfrac{8}{1\,200}}\right]$	✓ A = R920 667,815 ✓ correct substitution in correct formula /korrekte vervanging in korrekte formule
	F= R358 421,8873	
	Balance of the loan = R 920 667,815 - R35 8421,88734 119	✓ difference/verskil
	=R 562 246	
	ALTERNATIVE/ALTERNATIEF	✓ balance/balans (5)
	$P = \dfrac{x[1-(1+i)^{-n}]}{i}$	**OR**
	$P = 8\,842,16\left[\dfrac{1-\left(1+\dfrac{8}{1\,200}\right)^{(-83)}}{\dfrac{8}{1\,200}}\right]$	✓ correct formula /korrekte formule ✓ $n = -83$ ✓ 8 842,16
	Balance of the loan: P = R562 246	✓ correct substitution ✓ balance/balans (5)

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QUESTION 7/VRAAG7

Penalty for notation in 7.1 or 7.2- only once.

7.1		
	$\lim_{h\to 0} \dfrac{f(x+h)-f(x)}{h}$	✓ correct formula
	$= \lim_{h\to 0} \dfrac{3(x+h)^2 + 2 - 3x^2 - 2}{h}$	✓ subst. in formula
	$= \lim_{h\to 0} \dfrac{3(x^2 + 2hx + h^2) + 2 - 3x^2 - 2}{h}$	
	$= \lim_{h\to 0} \dfrac{3x^2 + 6hx + 3h^2 + -3x^2 - 2}{h}$	
	$= \lim_{h\to 0} \dfrac{6hx + 3h^2}{h}$	✓ simplifying
	$= \lim_{h\to 0} \dfrac{h(6x + 3h)}{h}$	✓ factorising
	$= \lim_{h\to 0}(6x + 3h)$	
	$= 6x$	✓ $6x$ (5)

7.2		
	$y = -(4x^2 - 12x + 9)$	✓ simplifying
	$y = -4x^2 + 12x - 9$	
	$\dfrac{dy}{dx} = -8x + 12$	✓ $-8x$ ✓ 12 (3)

7.3		
	$f(x) = \sqrt{x}\,(x+2)$	
	$f(x) = x\sqrt{x} + 2\sqrt{x}$	✓ simplifying
	$\quad = x^{\frac{3}{2}} + 2x^{\frac{1}{2}}$	
	$f'(x) = \dfrac{3}{2}x^{\frac{1}{2}} + x^{\frac{-1}{2}}$	✓ determining derivative of each fraction *lbepaal eerste afgeleide van elke term*
	$f'(4) = \dfrac{3}{2}(4)^{\frac{1}{2}} + (4)^{\frac{-1}{2}}$	✓ substitution of 4
	$\quad = 3 + 2^{-1}$	✓ answer (4)
	$\quad = 3\dfrac{1}{2}$	

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QUESTION 8/VRAAG 8

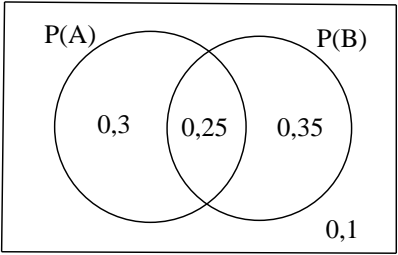| 8.1	$g(x) = (x+1)(x+1)(x-5) = 0$ $\therefore x = -1$ or $x = 5$ A$(-1\,;\,0)$ and D$(5\,;\,0)$	✓ A(-1; 0) ✓ D(5 ; 0) (2)
8.2	$g'(x) = 3x^2 - 6x - 9 = 0$ $\div 3 : x^2 - 2x - 3 = 0$ $(x-3)(x+1) = 0$ $x = 3$ or $x = -1$ $g(3) = -32$ $\therefore$ C$(3;-32)$ and A$(-1;0)$	✓ first derivative $= 0$ /*eerste afgeleide* ✓ C(3;-32) ✓ A(-1;0) (3)
8.3	EC: $y = -32$ $\therefore -32 = x^3 - 3x^2 - 9x - 5$ $0 = x^3 - 3x^2 - 9x - 5 + 32$ $0 = x^3 - 3x^2 - 9x + 27$ $0 = (x-3)(x-3)(x+3)$ $\therefore x = 3$ or $x = -3$ $\therefore x -$ coordinate at E $: x = -3$	✓ equation of tangent /*vergelyking van raaklyn* ✓ equating /*gelykstelling* ✓ value of x /*waarde van* (3)
8.4.1	$g'(x) = 3x^2 - 6x - 9$ $g''(x) = 6x - 6 = 0$ $\qquad x = 1$ $g(1) = (1)^3 - 3(1)^2 - 9(1) - 5 = -16$ $\therefore$ point of inflection: $(1;-16)$ OR $x = \dfrac{x_A + x_C}{2} \qquad y = \dfrac{y_A + y_C}{2}$ and $x = \dfrac{-1+3}{2} = 1 \qquad y = \dfrac{0-32}{2} = -16$ $\therefore$ point of inflection: $(1\,;-16)$	✓ second derivative=0 /*tweede afgeleide* ✓ value of x /*waarde van* ✓ value of y /*waarde van* OR ✓ average ✓ value of x ✓ value of y (3)
8.4.2	Gradient of tangent: $g'(1) = 3(1)^2 - 6(1) - 9 = -12$ Point of inflection $(1;-16)$ $\qquad\qquad y = -12x + c$ Equation of tangent: $\begin{array}{l} -16 = -12(1) + c \\ -4 = c \end{array}$ $\qquad\qquad \therefore y = -12x - 4$	✓ first derivative /*eerste afgeleide* ✓ value of gradient /*waarde van gradient* ✓ substitution of coordinates of the point of inflection ✓ equation of tangent (4)

QUESTION 9 /VRAAG9

9.1	Area shaded $= 8x - (2x)^2 - \pi\left(\dfrac{2}{3} \times 2x\right)^2$ $= 8x - \pi\left(4 + \dfrac{16}{9}\right)x^2$ $= 8x - \dfrac{52}{9}\pi x^2$	✓✓✓formula /*formule* ✓factorization (4) */faktorisering*
9.2	$A = 8x - \dfrac{52}{9}\pi x^2$ Maximum area: $D_x(A) = 8 - \dfrac{104}{9}\pi x = 0$ $\therefore 104\pi x = 72$ $x = \dfrac{72}{104\pi} = 0,22\,m$	✓ first derivative $= 0$ /*eerste afgeleide* ✓ simplifying /*vereenvoudiging* ✓ $_x$value /*waarde* (4)
9.3	maximum area $= 8(0,22) - \dfrac{52}{9}\pi(0,22)^2 = 0,88$ m^2	✓ answer /*antwoord* (1)

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QUESTION 10/VRAAG 10

10.1	For an odd product,the two numbers must be odd. *Vir 'n onewe produk moet albei getalle onewe wees* First draw: 4 odd out of 7 cards *Eerste trekking: 4 onewe van 7 kaarte* Second draw: 3 odd out of 6 *Tweede trekking: 3 onewe van 6* P(odd product) $= \dfrac{4}{7} \times \dfrac{3}{6} = \dfrac{2}{7} \approx 0,3$ P(*onewe produk*)$= \dfrac{4}{7} \times \dfrac{3}{6} = \dfrac{2}{7} \approx 0,3$	✓$\dfrac{4}{7}$ ✓$\dfrac{3}{6}$ ✓$\dfrac{2}{7}$ 0r 0,3 (3)

10.3		✓ 0,3 and 0,35 ✓ 0,25 ✓ 0,1 (3)
10.4.1	$P(A \text{ and } B^/) = 0{,}55 - 0{,}25 = 0{,}3$	✓ 0,55-0,25 ✓ 0,3 (2)
10.4.2.	$P(A \text{ or } B^/) = 1 - 0{,}35 = 0{,}65$	✓ answer /*antwoord* (1)

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QUESTION 11/VRAAG 11

11.1	$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1 = 720$	✓✓ answer /*antwoord* (2)
11.2	$2! \times 5! = 2 \times 120 = 240$	✓ $2!_{\times} 5!$ ✓ 240 (2)
11.3	Left/*links* : $1 \times 5!$ Right /*regs*: $5! \times 1$ $\dfrac{1 \times 5! + 5! \times 1}{6!}$ $\dfrac{120 + 120}{720}$ $= \dfrac{240}{720}$ $= \dfrac{1}{3}$ OR Left/*links* : $\dfrac{1}{6}$ Right /*regs*: $\dfrac{1}{6}$ $\dfrac{1}{6} + \dfrac{1}{6}$ $= \dfrac{2}{6} = \dfrac{1}{3}$	✓ $1 \times 5!$ ✓ $1 \times 5!$ ✓ $6!$ ✓ answer /*answer* OR (4)

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TOTAL 150